MTH5103 Complex Variables

Week 11 Practice Exercies

These exercises are for your daily practice.

1. Let f be holomorphic on and inside a circle C of radius R, centred at z_0. Use Gauss' Mean Value Theorem to deduce that either $|f(z)| > |f(z_0)|$ for some z on C or $|f(z)| = |f(z_0)|$ for all z on C. Conclude that f must be a constant function.

2. Use the previous exercise to prove the Maximum Modulus Principle.

3. State a *Minimum Modulus* Principle and modify the proof of Max-Mod to prove your statement.

4. Use Liouville's Theorem to prove that if f and g are entire functions and $|f(z)| \leq |g(z)|$ for all $z \in \mathbb{C}$, then $f = \alpha g$ for some constant α.

5. Prove the following claim: Suppose f is entire. If there exists a real constant $M > 0$ such that $|f(z)| \leq M|z|$ for all $z \in \mathbb{C}$, the f must be linear.

6. (Optional, harder problem for students interested/registered in Analysis) The image of an entire function is dense in $\mathbb{C}$.