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Problem Set 9.

1) a) Compute $\int_C (z^3 + z\bar{z})\,dz$ where $C = \{z : |z| = 1, -\frac{\pi}{2} \le \arg z \le \frac{\pi}{2}\}$

b) compute $\int_C e^{2z}\,dz$ where C is the straight line from $z_0 = 0$ to $z_1 = \pi(1+i)$.

c) $\int_C (z^2 - 1)\,dz$, where $C = \{z : |z-1| = 1\}$

d) $\int_C ((x-y) + i(x+y))\,dz$, where $C = \{z : |z-1| = 1\}$

2) Using the ML inequality prove:

a) $\left| \int_C \frac{dz}{z^2-1} \right| < \frac{\pi}{3}$, where C is an arc of the circle $|z| = 2$ starting at $z = 2$, ending at $z = 2i$ in the first quadrant

b) $\left| \int_C \frac{e^{2z}}{6z^5}\,dz \right| \le \frac{\pi e^2}{3}$, where C is the unit circle traversed once anticlockwise.

3) a) Let $C = \{z : |z| = 1\}$. Prove that $\int_C f(z)\,dz = 0$, where

i) $f(z) = \frac{z^3}{z+1-2i}$

ii) $f(z) = ze^{-z}$

iii) $f(z) = \frac{1}{z^2+3z+4}$

iv) $f(z) = \mathrm{Ln}(z-4)$

v) $f(z) = \cos z$

State the relevant version of the Cauchy Theorem.

b) Let $f(z) = \frac{z^2}{z-1+i}$

i) Compute $\int_{C_1} f(z)\,dz$, where $C_1 = \{z \in \mathbb{C} : |z + 3i - 5| = 1\}$

ii) Compute $\int_{C_2} f(z)\,dz$, where $C_2 = \{z \in \mathbb{C} : |z| = 3$

4) Compute the following integrals

a) $\int_{|z|=5} \frac{dz}{z^2+5z+4}$

b) $\int_{|z+2i|=1} \frac{\sin z}{z^3+16z}$

State all the theorems/propositions that you use during computation.