MTH5103 Complex Variables

Week 10 Practice Exercies

These exercises are for your daily practice.

1. Give examples of the following: a convex domain, a star-shaped domain, a star-shaped domain which is nonconvex. Include both a drawing and a definition of your set.

2. Read and check the details for Theorem 2 in the Week 10 Lecture Notes, Cauchy's Theorem for a Triangle.

3. Verify $\int_C \frac{z}{(z-4)^2} dz = 2\pi i$, where C is the contour defined by traversing once the square with vertices $\pm 3i, 6 \pm 3i$, anti-clockwise. Do *not* use the Deformation Principle (as we did in class).

4. In the proof of the Residue Theorem, the last line claims that $\text{Res}(f; z_k) = \varphi(z_k)$. Confirm this using results from your Week 8 Lecture Notes. Note that z_k is a simple pole, so $f(z) = \varphi(z)/(z - z_k)$.

5. Classify the singularities of $\frac{z}{(z-4)^2}$ and use this to calculate $\text{Res}\left(\frac{z}{(z-4)^2}; 4\right)$.

6. Let $f(x, y)$ be a *real* function of two real variables. Determine the conditions on f in order for the following equality to hold:

$$\frac{\partial}{\partial y} \int_a^b f(x, y) dx = \int_a^b \frac{\partial}{\partial y} f(x, y) dx.$$

   What if a and b are functions of y?

7. Use the Residue Theorem to compute $\int_C \frac{z^3}{(z-1)^4}$, where C is any simple closed positively oriented contour around $z = 1$.