# Problem Set 8.

1) a) Find the zeros and their order for the following functions:

1) $z^3 \cos z$ 2) $\cos(z^3)$ 3) $\cos z + \sinh(iz)$

b) Find the order of $z=0$ for the following functions:

1) $\frac{z^5}{1+z-e^z}$ 2) $6\sin(z^3)+z^9-6z^3$ 3) $(e^{z^2}-1-z^2)\sin^3 z$

c) If $z_0$ is a zero of order $n$ of an analytic function $f(z)$ and it is zero of order $m$ of an analytic function $g(z)$, what can be said about $z_0$ for the following functions:

1) $f(z)+g(z)$ 2) $f(z)g(z)$ 3) $\frac{f(z)}{g(z)}$

d) Prove: If $z=z_0$ is a zero of order $n$ of $f(z)$, then it is zero of order $2n$ of $(f(z))^2$.

2) a) Write the following curves in parametric way $z(t)=x(t)+iy(t)$

i) $z\bar{z}+i(z-\bar{z})-2=0$ ii) $\operatorname{Im}\left(\frac{1}{z}\right)=\frac{1}{2}$

b) Prove that $\operatorname{Im} z=0$, $|\arg z-\frac{\pi}{2}|=\frac{\pi}{2}$, $z-\bar{z}=0$, $|z-i|=|z+i|$ are different equations all describing the real $(x)$ axis.

3) Give an example of a function $f(z)$ which has the following properties. Justify your answer.

a) A pole of order 3 at $z_0=2i$ with residue $\frac{1}{5}$

b) A simple pole at $z_0=i-3$ with residue $8(i-3)$, and an essential singularity at $z_1=i$ with residue 6.

c) A pole of order 4 at $z_0=4i-1$ with residue 5, and a simple pole at $z_1=-i$ with residue $-\frac{3}{4}$.

4) Let the curve $C$ be given by the graph of the function $y=f(x)$ with $f(x)=\cosh x$, $0\le x\le 1$

a) Draw a sketch of the curve

b) Give a path $\gamma:[0,1]\to\mathbb{C}$ which has the curve $C$ as its image

c) Compute the length $L$ of the curve

5) Using the definition of the contour integral compute the integral of $f(z)=1+i-2\bar{z}$ from $z_0=0$ to $z_1=1+i$ along the following contours:

a) Straight line segment

b) Parabola $y=x^2$

c) Broken line $z_0, z_2, z_1$ with $z_2=1$.