

MTH5103 Complex Variables

Week 9 Practice Exercises

These exercises are for your daily practice.

1. Consider the path $\gamma_3 : [0, 1] \rightarrow \mathbb{C}$ defined by $\gamma_1(t) = e^{4\pi it}$. Graph the curve represented by this parametrisation. How does this compare with γ_1 and γ_2 from the first Example of the Week 9 Lecture Notes?
2. Give a parametrisation for the curve represented by a square with vertices $(\pm 1, \pm 1)$, traversed once anticlockwise.
3. Parametrise a curve starting at $z = 0$ and ending at the point $z = 4$ which passes through the upper semicircle of the unit circle centred at 1 and passing through the lower semicircle of the unit circle centred at 3. First graph the circles and trace out the curve to be parametrised. Hint: Use the sum of two parametrisations, one for each semicircle.
4. Calculate the length of the square contour found in exercise # 2. Repeat this exercise for #3.
5. Let C be a semicircle parametrised by the path $\gamma(\theta) = 3e^{i\theta}$, $0 \leq \theta \leq \pi$ and let $f(z) = z^3 + 7z + 4$. Calculate $\int_C f(z)dz$.
6. Using the ML Inequality, estimate the integral $\left| \int_C f(z)dz \right|$ for each of the functions and contours in the previous exercises.