Problem Set 7.

1) a) Compute: 1) $\mathrm{Ln}(-i)$ 2) $\mathrm{Ln}(i^i)$ 3) $\mathrm{Ln}(3-2i)$

b) Find the branch points:

1) $\mathrm{Ln}(z^4-2)$ 2) $(z-8i)\,\mathrm{Ln}\left(\frac{3z-i}{z+4}\right)-4i$

c) Prove: The function $f(z)=\frac{\mathrm{Ln}(z-1)}{z^2+i}$ is analytic in the cut-plane with the cut along the negative half of the real axis up to (including) $x=1$ and punctured at the points $z=\pm\frac{1}{\sqrt{2}}(1-i)$

2) a) Prove: $\arccos z=-i\,\mathrm{Ln}(z\pm i\sqrt{1-z^2})$; $\mathrm{arctg}\, z=\frac{1}{2i}\mathrm{Ln}\left(\frac{1+iz}{1-iz}\right)$

b) Compute: 1) $\arcsin\left(\frac{\pi}{3}i\right)$ 2) $\mathrm{arctg}\,(1+i)$

c) Solve the following equations: 1) $\sin z=5$ 2) $3\cos z-7=0$

d) Define (similarly to what we did in class) the inverse hyperbolic functions $\mathrm{arcsinh}\, z$, $\mathrm{arccosh}\, z$

e) Prove: 1) $\mathrm{arcsinh}\, z=\mathrm{Ln}(z\pm\sqrt{z^2+1})$, 2) $\mathrm{arccosh}\, z=\mathrm{Ln}(z\pm\sqrt{z^2-1})$

3) a) Classify the singularities of $f(z)=\frac{\sin(z-1)}{z^3+z^2-z-1}$. Justify your answer. Compute the residue at each singularity.

b) Find and classify the singularities of:

1) $\frac{z^2-1}{z^6+2z^5+z^4}$ 2) $\frac{z-\sin z}{z^7}$ 3) $\frac{e^{1/z}-1}{e^z-1}$ 4) $\cosh^2\frac{1}{z-\pi}$

5) $\frac{\sinh(z^4)-z}{z^6}$ 6) $z^5\cosh\frac{1}{z^2}$ 7) $\frac{e^{-iz}}{z^2-\frac{\pi^2}{4}}$ 8) $\frac{1-\cosh z}{z^5}$

c) Compute the residue of each function from b) at each singularity.

4) Consider the function $f(z)=\frac{1}{(z-4)(z+8i)}$

a) Find the Taylor series of f on a disc centered at $z_0=1$. Determine its radius of convergence

b) Find the Laurent series of f on a punctured disc centered at $z_0=-8i$. Where is this series valid? (absolutely convergent?) What is the principal part of this series? What type of singularity does f have at $z_0=-8i$? What is the residue of f at $z_0=-8i$?