

# MTH5103 Complex Variables

## Week 8 Practice Exercises

*These exercises are for your daily practice.*

1. Why does the function  $f(z) = \frac{1}{(z-1)(z-2)}$  have isolated singularities at  $z = 1$  and  $z = 2$ ?
2. How does the function  $f(z) = e^{1/z}$  demonstrate that the “single point exception” is needed in the Great Picard Theorem?
3. Calculate  $\text{Res}\left(\frac{e^z - 1}{z}; 0\right)$ .
4. Calculate  $\text{Res}\left(\frac{z+1}{z^2+9}; -3i\right)$ .
5. Prove that  $f$  has a zero of order  $m$  at  $z_0$  if and only if  $f(z_0), f'(z_0), \dots, f^{(m-1)}(z_0)$  are all equal to zero, but  $f^{(m)}(z_0) \neq 0$ .
6. At the end of the Proposition 2, in the Week 8 Lecture Notes, why can we conclude the last sentence, that  $f$  has a pole of order  $m$  at  $z_0$ ?
7. Compute  $\text{Res}\left(\frac{1}{z(e^z - 1)}; 0\right)$ .