MTH5103 Complex Variables

Week 6 Practice Exercies

These exercises are for your daily practice.

1. Consider the function $f(z) = \dfrac{1}{2z-3}$.
   (a) Find the power series expansion of $f(z)$ about the point $z_0 = 0$ and determine the radius of convergence. *Hint:* $R = \frac{3}{2}$.
   (b) Find the power series expansion of $f(z)$ about the point $z_0 = 2$ and determine the radius of convergence. *Hint:* $R = \frac{1}{2}$.
   (c) Find the power series expansion of $f(z)$ about the point $z_0 = \frac{3}{2} + i$ and determine the radius of convergence. *Hint:* $R = 1$.
   (d) Draw the various discs of convergence in each of (a) - (c) and discuss your answers in the context of the results we have stated on power/Taylor series.

2. Consider the function
$$f(x) = \begin{cases} e^{-1/x} & x > 0 \\ 0 & x \leq 0 \end{cases} . \tag{1}$$
   Graph/sketch this function. Show that this function satisfies $f^{(n)}(0) = 0$ for all $n \geq 0$ so its Taylor series must be $0 + 0 \cdot x + 0 \cdot x^2 + \cdots = 0$. Why is f itself is <u>not</u> equal to its Taylor series?

3. When finding the Laurent series of $\dfrac{1}{1-z}$ to equal $\displaystyle\sum_{n=1}^{\infty}(-1) \cdot \frac{1}{z^n}$, why is the point $z = 0$ not a problem?

4. Draw the annulus $A = \{z : 0 < |z-1| < 1\}$. What is the Laurent series of $f(z) = \dfrac{1}{(z-1)(z-2)}$ on A? *Do this exercise without looking at the notes!*