Recommended Revision Tips

1. Finding fixed points of (low degree) polynomials – quadratics, cubics

2. Determining if fixed points are attracting or repelling (with justification)

3. Basin of attraction – determining basin of attraction of an attracting fixed point <u>OR</u> be able to give an example of an f with a given interval as its basin of attraction.

4. Finding eventually fixed points which are not themselves fixed points.

<u>OR</u> " " periodic " "

" " " periodic " .

5. Have a strategy for deciding and justifying if two functions f and g are topologically conjugate:

(i) Do they have the same number of fixed points? If not then f, g are not topologically conjugate.

(ii) If they do have the same number of fixed points then they might be top. conj. – you could test this by assuming the conjugacy takes the form $h(x) = ax + b$, then checking whether the conjugacy equation $hf(x) = gh(x)$ is satisfied for some values of a and b.

[Doing this may involve solving 2 simultaneous equations in the ~~'variables'~~ 'unknowns' a and b]

6. <u>Diffeomorphisms</u>

- Know the theoretical results proved
- Be able to give examples (e.g. of a diffeomorphism with some $\{a, b\}$ as a period-2 orbit)

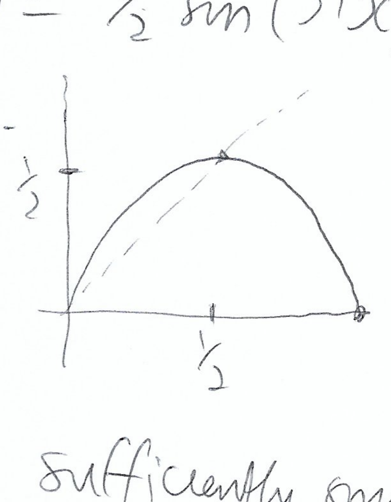
7. <u>Parametrised families</u>

In lectures we looked at $f_\lambda(x) = \lambda x(1-x)$ for $\lambda \in [0, 4]$

However we also <u>briefly</u> mentioned

$$f_\lambda(x) = \lambda \sin(\pi x).$$

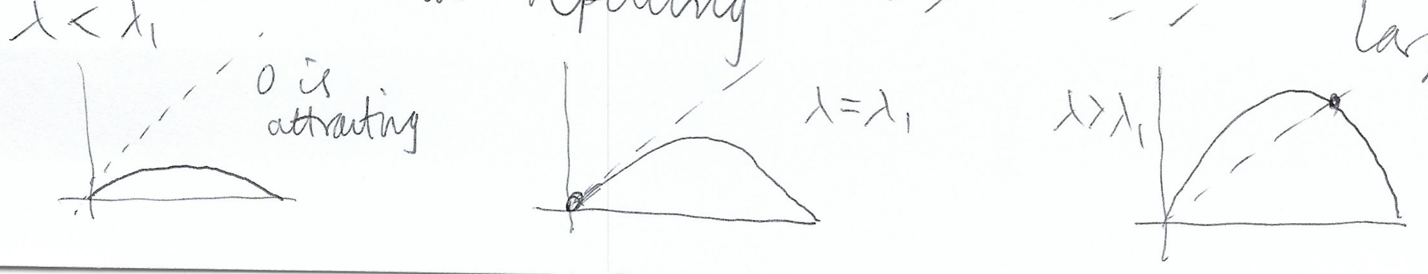
e.g. The case $\lambda = \frac{1}{2}$, $f_{\frac{1}{2}}(x) = \frac{1}{2}\sin(\pi x)$ arose in the Week 11 Test.



Properties: 0 is a fixed pt for <u>all</u> λ

: 0 is attracting if λ is sufficiently small

and repelling " " " large



If λ_1 is such that $f'_{\lambda_1}(0) = 1$ then λ_1 is the 'transition[*] value'

8. Sensitive Dependence on Initial Conditions (SDIC)

— What does it mean? (Technical definition)

— Which f do/don't have SDIC?

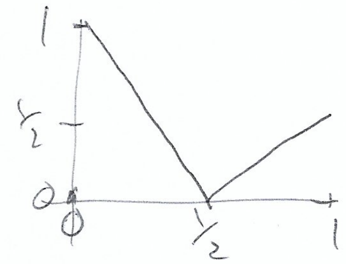
(Intuition → Proof)

9. 'Piecewise defined' functions, in particular 'piecewise linear' functions

e.g. In the Test, $f(x) = \begin{cases} 1-2x, & x \in [0, \tfrac{1}{2}) \\ x - \tfrac{1}{2}, & x \in [\tfrac{1}{2}, 1] \end{cases}$

- Be able to <u>draw their graphs</u>

(Key point: Evaluate f at the <u>endpoints</u> of the intervals of definition)



- Be able to find fixed points (drawing the graph is a useful first step!)

- Be able to <u>determine orbits</u> – this involves knowing when to use the various 'pieces' in the definition of the function

  e.g. $\mathcal{O}(\tfrac{1}{2}) = \{\tfrac{1}{2}, 0, 1\}$

- Be aware that I may use piecewise defined functions to test knowledge of <u>Sharkovskii's Theorem</u>

10. Non-escaping points
    - What are they are?
    - Connection with fractals, and iterated function systems

11. Iterated function systems : $\varphi_1, \varphi_2, \ldots, \varphi_l$; Φ
    - Be comfortable with the definition
    - Be familiar with examples, especially in Euclidean space $\mathbb{R}^d$ where $d = 1$ or 2

Be able to compute $\Phi([0,1])$

or $\Phi([0,1]^2)$

- Pay attention to 'Scaling factors' (box side length, and number of boxes)
- Be able to apply to computing box dimension