Complex Variables 2019 Sample exam.

Q1 (20)

a) Solve the following equation

$$(z-4i)(3i-5) = (2i-z)(1+i)$$

Express the solution in Cartesian $(x+iy)$ form. Justify all of your steps (10)

b) Describe graphically the set of points in the complex plane defined by the following equation

$$|z| = \operatorname{Re} z + \frac{1}{4} \qquad (10)$$

Q2 (20)

a) State the Root (Cauchy) Test for complex series (7)

b) Using the Root Test or otherwise, determine the radius of convergence of the power series

$$\sum_{n=0}^{\infty} (1+i)^n z^n \qquad (10)$$

c) Does the series converge for any z with $|z| = \frac{1}{\sqrt{2}}$? (3)

Q3 (20)

a) Find the coefficients a_n and b_n of the Laurent series

$$\sum_{n=0}^{\infty} a_n (z-z_0)^n + \sum_{n=1}^{\infty} b_n (z-z_0)^{-n}$$

of $f(z) = \dfrac{1}{(z-4)(z+2)}$ on a punctured disc centered at $z_0 = 4$ (10)

b) Determine the residue of $f(z)$ at the point $z = 4$ (10)

Q4 (20)

a) Prove that the function $u(x,y) = 4e^x \sin y$ is harmonic and find its harmonic conjugate (10)

b) Find all the singularities of the function $f(z) = \dfrac{e^{-iz}}{z^2 - \frac{\pi^2}{9}}$ and determine the nature of these singularities. (10)

Q5 (20)

a) State the Residue Theorem (10)

b) Using the Residue Theorem or otherwise, compute

$$\int_C \frac{z-1}{(z+3)(z-6)^3}\, dz$$

where C is positively oriented circle of radius 3 centered at $z = 5$ (10)