Queen Mary
University of London

Late-Summer Examination period 2018

MTH5103: Complex Variables

Duration: 2 hours

Write your solutions in the space provided in this exam paper.

Student number: | | | | | | | | | |

Desk number: | | | |

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately.

It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms, it shall be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiners: M. Shamis

 Turn Over

This page is for marking purposes only:

DO NOT WRITE ON IT

Question	Mark	Subpart Breakdown
1		
2		
3		
4		
5		
TOTAL :		

Question 1. [20 marks]

(a) Compute the number

$$\left|\frac{(3+4i)(1+i)^6}{i^5(2+4i)^2}\right|$$

and express it in Cartesian $(x+iy)$ form. Justify all of your steps. **[10]**

Write your solution to Question #1(a) below

(b) Find and plot all the solutions $z \in \mathbb{C}$ of the equation $z^5 = 32$. **[10]**

Write your solution to Question #1(b) below

Additional space for Question 1

Question 2. [20 marks]

(a) State the Root (Cauchy) Test for complex series. **[10]**

Write your solution to Question #2(a) below

(b) Using the Root Test, or otherwise, determine the values of z for which the power series

$$\sum_{n=1}^{\infty} \left(\frac{z}{in}\right)^n$$

converges. What is the radius of convergence? **[10]**

Write your solution to Question #2(b) below

Question 3. [20 marks]

(a) Find the coefficients a_n and b_n of the Laurent series

$$\sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^{-n}$$

of $f(z) = \frac{1}{(z-6)(z+3)}$ on a punctured disc centered at $z_0 = -3$. **[10]**

Write your solution to Question #3(a) below

(b) Where is the series from (a) absolutely convergent? **[10]**

Write your solution to Question #3(b) below

Additional space for Question 3

Question 4. [20 marks]

(a) Prove that the function $f(z) = |z|^4$ is differentiable at $z = 0$ but not differentiable anywhere else. **[10]**

Write your solution to Question #4(a) below

(b) Find an analytic function $f(z),\ z = x + iy$ such that

$$\mathrm{Re}\, f(z) = x^2 - y^2 + 2x \quad \text{and} \quad f(i) = 2i - 1.$$

[10]

Write your solution to Question #4(b) below

Additional space for Question 4

Question 5. [20 marks]

(a) Find all singularities of the function

$$f(z) = \frac{z - \sin z}{z^4},$$

and determine the nature of each of these singularities. **[10]**

Write your solution to Question #5(a) below

(b) Compute the integral

$$\int_C \frac{z - \sin z}{z^4} dz,$$

where C is the positively oriented circle of radius 2 centered at the origin. You may use the result of (a). **[10]**

Write your solution to Questions #5(b) below

Additional space for Question 5

This page is for additional work and will NOT be marked.

End of Paper.