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Problem Set 2.

1) Using Euler's formula prove the following:
   a) For any $z \in \mathbb{C}$ $\sin^2 z + \cos^2 z = 1$
   b) For any $z_1, z_2 \in \mathbb{C}$ $\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$
   c) $\cos z = 0$ if and only if $z = \frac{\pi}{2} + \pi k$, $k \in \mathbb{Z}$
2) Find all the solutions of the following equations (if there is no solution write- no solution) Justify your answer
   a) $e^z - i = 0$ b) $e^z = 1$ c) $e^{iz} + 1 - i = 0$
3) a) Find the real and imaginary parts of the following functions:
   1) $f(z) = \frac{z-1}{z+1}$
   2) $f(z) = 3z^2 + \frac{1}{z}$

   b) Rewrite the following functions in terms of $z, \bar{z}$:
   1) $f(x,y) = x + 2y + i(3y - x)$
   2) $f(x,y) = x^2 + y^2 - i(4x - y)$
4) a) Find the image of the following under $w = z^2$
   1) The line $x = 3$
   2) The domain $2 < y < 3$

   b) Let $f(z) = \frac{1}{z}$
   1) What is the image of the line $y = 2$ under f?
   2) Show that $w = \frac{1}{z}$ maps the interior of the unit circle in the first quadrant to the exterior of the unit circle in the fourth quadrant
5) Prove:
   a) $\lim_{z \to 1-2i} \frac{1}{z+2i} = 1$
   b) $\lim_{z \to 0} \frac{(\operatorname{Im} z)^2}{|z|^2}$ does not exist
   c) $\lim_{z \to 0} \frac{(\operatorname{Re} z)^2}{|z|} = 0$
   d) Compute $\lim_{z \to 3i} \frac{z^2 - 4i}{2z^2 + 2i}$