

Problem set 1.

1) a) Find the real numbers a, b if

$$4a + (3-2b)i = 13+3i \quad 4(5-2a) - (b-4)i = 10-5i$$

b) Compute the following numbers

$$(-10-2i)(4+5i); (-3+7i)^2 - (2+i)^2; (i+i^2+\dots+i^{157})^{37}$$

c) Solve the following equations (find the roots)

1) $(2z-i)(3+2i) = (z+i)(1-3i)$

2) $\frac{1-iz}{1+iz} = -2i$

3) $i(3z-1) = 3z\bar{z}$

d) Prove that the following numbers are real (for any $z \in \mathbb{C}$)

$$\frac{1}{z} + \frac{1}{\bar{z}}; \quad z^3\bar{z} + z\bar{z}^3$$

e) Compute the following (express it in the form $x+iy$)

$$\left| \frac{(3+5i)(1-2i)^6}{i^{20}(3+5i)^2} \right|$$

2) Express the following complex numbers in polar form

a) $-1+i$ b) $\sqrt{3}-i$ c) 4 d) $-i$ e) $\frac{-2}{1+i\sqrt{3}}$ f) $5+3i$

3) Find all the roots of the following equations

a) $z = \sqrt[7]{1-i}$

b) $z^{14} = i$

c) $z = \sqrt[6]{64}$

4) Describe graphically the sets of points in the complex plane defined by the following equations and inequalities

a) $|z-4i| > 2$

b) $\operatorname{Re} z^2 = 4$

c) $\operatorname{Re}\left(\frac{1}{z}\right) = \frac{1}{8}$

d) $|z| = \operatorname{Re} z + \frac{1}{3}$

e) $|z+1| + |z-1| = 4$