MTH5103 Complex Variables

Week 1 Practice Exercies

These exercises are for your daily practice.

1. Prove Proposition 1 from the Lecture Notes, Week 1.

2. Let $z_1 = 5 + i, z_2 = 3 - i$. Calculate $z_1 z_2$, $\dfrac{z_1}{z_2}$, $z_1 \bar{z}_2$.

3. Use the definition of complex conjugation to prove $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$.

4. Why is $\Re z \leq |z|$? Is it true that $\Re z + \Im z \leq |z|$ (why or why not)?

5. Calculate the fifth roots of unity, i.e., find $z \in \mathbb{C}$ such that $z^5 = 1$.

6. Fill in the diagrams for each of the four examples in the Lecture Notes, Week 1. In particular,

   (a) Example 1: Graph $z_1 = 1 + i$ and $z_2 = 2 - i$ on the complex plane as well as $z_1 + z_2$, $z_1 z_2$, and z_2^{-1}. Verify that $z_1 + z_2$ represents vector addition in $\mathbb{R}^2$.

   (b) Example 2: Pick a point z on the unit circle in the first quadrant of the complex plane $\mathbb{C}$. Where should $\dfrac{1}{z}$ be? Use the discussion in this example to interpret $\dfrac{1}{z}$ for values z inside and outside the unit circle.

   (c) Example 3: Let $w = 1$. Draw the nth roots of w for $n = 1, 2, 3, 4, 5, 6$. How do your diagrams change if $w \neq 1$?

   (d) Example 4: Verify that the four values we obtain in modulus-argument form for $k = 0, 1, 2, 3$ indeed correspond to the listed points $\pm(1 \pm i)$ and graph these on the circle of radius $\sqrt{2}$, as we did in class.