MAS205 Complex Variables 2006-2007

Calculus I and II revision class

Exercise 1. Describe graphically the set of points $(x, y) \in \mathbb{R}^2$ which satisfy the inequality $x^2 + y^2 \leq 3 + 2x$.

Exercise 2. Evaluate $\sum_{n=0}^{\infty} \frac{3^n}{4^n}$.

Exercise 3. Which of the following series converges?

$$(a) \quad \sum_{n=1}^{\infty} \frac{3}{2n-1} \qquad (b) \quad \sum_{n=0}^{\infty} \frac{n^2+2}{7-n^2} \qquad (c) \quad \sum_{n=1}^{\infty} \frac{n}{2^n}$$

Exercise 4. Find the points $x \in \mathbb{R}$ at which the function $f(x) = \dfrac{\sin x}{1 + \cos x}$ is differentiable, and calculate its derivative at these points.

Exercise 5. Find thre first three terms of the Taylor series of $f(x) = \cos x$ about the point $\pi/3$.

Exercise 6. Let f and g be functions from $\mathbb{R}^2$ to $\mathbb{R}$ which vanish at the origin $(0, 0)$, and, elsewhere, are given by

$$f(x, y) = \frac{x^2 - y^2}{(x + y)^2}\,, \quad g(x, y) = \frac{xy^2}{x^2 + y^2}\,.$$

Determine whether f and g are joint continuous at $(0, 0)$.

Exercise 7. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x, y) = e^x \cos(x + y)$. Find $\dfrac{\partial}{\partial x} f$ and $\dfrac{\partial}{\partial y} f$.