MTH6112 Actuarial Financial Engineering
Coursework Week 9

1. Assume that the risk-free interest rate is governed by the Vasicek model. Historical data of short time risk-free interest rate is given in the table for a period January-May 2019

Date	01/01	01/02	01/03	01/04	01/05
r_t	3.56%	4.02%	3.84%	4.00%	4.18%

There are three zero-coupon bonds (see the table for their parameters) available at the market paying £1 on a corresponding maturity day.

	Issue date	Maturity date	Price on issue date
Bond 1	01/01	01/03	£0.92
Bond 2	01/02	01/04	£0.86
Bond 3	01/03	01/05	?

Find the price of Bond 3.

2. Let W_t be a standard Brownian Motion. The simplest version of the Ornstein-Uhlenbeck process X_t is defined by

$$X_t = \mathrm{e}^{-\theta t} W_{\mathrm{e}^{2\theta t}}, \quad \text{for some constant } \theta > 0.$$

a) Does this process have independent increments?

b) Is X_t a Brownian Motion?

c) What is the distribution of the increment $X_t - X_s$ for $t > s$?

d) Compute $\mu_m := \mathbb{E}[(X_t)^m]$ for all integer $m > 0$.

e) Compute Cov $\{X_t, X_s\}$.