MTH6112 Actuarial Financial Engineering
Coursework Week 8

1. (a) State the definition of the 'usual' differential of a function $F(x)$.

   (b) Use the relation $F(x+dx) - F(x) \approx dF(x)$ it to compute $e^{0.1}$ and $\ln 0.9$ (without a calculator).

   Then compute $e^{0.1}$ and $\ln 0.9$ using a calculator and compare the results.

2. Reed Section 13.2 in the Slides of Week 8. Pay attention to the examples.

   (a) State Ito's lemma for $F(W_t)$.

   (b) Compute $dF(W_t)$ for $F(x) = x^3$ and for $F(x) = e^x$.

   (c) Use the results obtained in (b) to compute the following integrals

   $$\int_0^t (W_s)^2 dW_s \quad \text{and} \quad \int_0^t e^{W_s} dW_s.$$

3. a) Consider a diffusion process $V(t)$, $t \geq 0$, satisfying the following stochastic differential equation:

   $$dV_t = (a + b\cos t)V_t dt + \sigma V_t dW_t. \tag{1}$$

   Solve this equation using the method explained in lectures.

   b) Write down $V(t)$ with $V_0 = 2.8$, $a = 0.5$, $b = 0.1$, and $\sigma = 0.2$.