MTH6112 Actuarial Financial Engineering
Coursework Week 1

You are **not supposed to submit solutions**. However, solving all the questions would help you to prepare for the exam. So, try to solve as many of them as you can. We shall discuss them during the tutorial.

1. Compute the expectation and the variance of a Brownian motion Y_t with drift parameter μ and volatility parameter σ.

   Remark. We often use the notation Y_t instead of $Y(t)$. Similar convention applies to other random processes.

2. In lectures, we have shown that $\mathrm{Cov}(W_t, W_s) = \min(t, s)$ (here W_t is the Wiener process). Using this result (or otherwise), compute the covariance $\mathrm{Cov}(Y_t, Y_s)$ of a Brownian motion with drift parameter μ and volatility parameter σ.

3. Let $W(t)$ be a Wiener process. For a real number $a > 0$, define a new process

$$Z(t) = \sqrt{a}W(at).$$

   Prove that $Z(t)$ is again a Wiener process.

4. A share price $S(t)$ evolves according to a Geometric Brownian motion with drift parameter μ and volatility parameter σ. Compute the moments $\mathbb{E}(S(t)^m)$ for all m (which may not be an integer number). Also, compute the variance of $S(t)$.

   Remark. To do that, you don't have to repeat the calculations we have carried out in the lecture.