Queen Mary
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MTH5126 Statistics for Insurance

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Week 11

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Run-off triangles (continued)

The average cost per claim method

- Assumptions underlying the ACPC method
- Some valid questions

Loss ratios

The Bornhuetter-Ferguson method

- Example

The average cost per claim method

- This method considers separately the two key elements of total claim amounts - namely the number of claims and the average amounts of claims.
- The method requires development tables for both total claim amounts and claim numbers.
- A third development table, of the average claim amounts, is then formed by dividing the figures in the corresponding cells of the first two tables.
- The next stage is the projection of figures in the average claims and number of claims tables, using either grossing up factors or development factors.

The average cost per claim method

- A grossing-up factor is not very different from a development factor. A grossing-up factor gives the proportion of the ultimate claim amount that has been paid so far.
- Suppose that we had cumulative payment amounts of:

$$500, 800, 1000, 1100$$

- The development factors for this row would be:

$$\frac{800}{500} = 1.6 \qquad \frac{1000}{800} = 1.25 \qquad \frac{1100}{1000} = 1.1$$

- The grossing-up factors would be calculated as follows:

$$\frac{500}{1100} = 45.45\% \qquad \frac{800}{1100} = 72.73\% \qquad \frac{1000}{1100} = 90.91\%$$

The average cost per claim method

- The projected ultimate claims can be calculated by multiplying together for each accident year the projected figures for the average claim amounts and claim numbers.
- A reserve can then be calculated by subtracting all payments to date in respect of claims relating to the data in the table.
- Let’s look at an example.

The average cost per claim method

	Development Year						
Accident Year	0	1	2	3	4	5	Ult
1	2,777	3,264	3,452	3,594	3,719	3,717	3,717
2	3,252	3,804	3,973	4,231	4,319		
3	3,725	4,404	4,779	4,946			
4	4,521	5,422	5,676				
5	5,369	6,142					
6	5,818						

This table shows cumulative incurred claims.

- The table shows cumulative incurred claims data, by years of accident and reporting development.

The average cost per claim method

Development Year

Accident Year	**0**	**1**	**2**	**3**	**4**	**5**	**Ult**
1	414	460	482	488	492	494	494
2	453	506	526	536	539		
3	494	548	572	582			
4	530	588	615				
5	545	605					
6	557						

This table shows cumulative number of reported claims.

- Number of reported claims, by year of accident and reporting development (Note that these are also cumulative).

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The average cost per claim method

- Dividing each cell in the first table by the corresponding cell in the second table gives the accumulated average incurred cost per claim.

	Development Year						
Accident Year	**0**	**1**	**2**	**3**	**4**	**5**	**Ult**
1	6.708	7.096	7.162	7.365	7.559	7.524	7.524
2	7.179	7.518	7.553	7.894	8.013		
3	7.540	8.036	8.355	8.498			
4	8.530	9.221	9.229				
5	9.851	10.152					
6	10.445						

This table shows average incurred cost per claim.

- These tables lead to the grossing-up factors and projected ultimate figures given in the next table.

The average cost per claim method

- The projections are based on the underlined simple averages of the grossing-up factors.

Development Year

Accident Year	0	1	2	3	4	5	Ult
1	6.708	7.096	7.162	7.365	7.559	7.524	7.524
	89.2%	94.3%	95.2%	97.9%	100.5 %	100.0%	
2	7.179	7.518	7.553	7.894	8.013		7.973
	90.0%	94.3%	94.7%	99.0%	<u>100.5 %</u>		
3	7.540	8.036	8.355	8.498			8.632
	87.4%	93.1%	96.8%	<u>98.45%</u>			
4	8.530	9.221	9.229				9.657
	88.3%	95.5%	<u>95.57%</u>				
5	9.851	10.152					10.766
	91.5%	<u>94.3%</u>					
6	10.445						11.699
	<u>89.28%</u>						

This table shows average incurred cost per claim.

The average cost per claim method

Let's see how these figures are calculated.

- Accident Year 1 is fully run off. We can express the figures for each year as a percentage of 7.524, the final figure. E.g.:

  $$7.365/7.524 = 0.979$$

- Next look at Accident Year 2. We use the corresponding figure in Accident Year 1 (because we have already filled in the percentages here) to find a grossing-up factor for Development Year 4 of 100.5%.

- So we can find the ultimate expected payout figure for Accident Year 2 as

  $$8.013/1.005 = 7.973$$

- Now calculate the grossing-up factors for Accident Year 2 by expressing the figures in Accident Year 2 as a percentage of 7.973.

The average cost per claim method

- Now look at Accident Year 3. We calculate the grossing-up factor for Development Year 3 by taking the average of the two figures that we already know, i.e.:

$$(97.9 + 99.0)/2 = 98.45\%$$

- Now calculate the ultimate figure for Accident Year 3:

$$8.498/0.9845 = 8.632$$

- We use this figure to calculate grossing-up factors for the whole of Accident Year 3.
- Continuing through the table we use the average of the known grossing-up factors to calculate the required grossing-up factor for each development year in turn.
- We end up with an ultimate claim amount for each accident year.
- We now do exactly the same for the claim number table.

The average cost per claim method

- The projections for the number of claims are based on the underlined simple averages of the grossing-up factors.

Development Year

Accident Year	0	1	2	3	4	5	Ult
1	414	460	482	488	492	494	494
	83.8%	93.1%	97.6%	98.8%	99.6 %	100.0%	
2	453	506	526	536	539		541
	83.7%	93.5%	97.2%	99.1%	99.6%		
3	494	548	572	582			588
	84.0%	93.2%	97.3%	98.95%			
4	530	588	615				632
	83.9%	93.0%	97.37%				
5	545	605					649
	84.0%	93.2%					
6	557						664
	83.88%						

This table shows number of claims.

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The average cost per claim method

- The total ultimate loss is therefore the sum of the following projected amounts for each accident year:

AY	Average cost per claim ×	Claim numbers =	Projected Loss estimate
1	7.524	494	3,717
2	7.973	541	4,313
3	8.632	588	5,076
4	9.657	632	6,103
5	10.766	649	6,987
6	11.699	664	7,768
	Total Projected Loss Estimate =		33,964

- If the claims paid to date amounted to £20,334 (assumed number) then the total reserve required would be £13,630.
- The triangle is based on incurred claims so we cannot deduce total paid claims from it.

The average cost per claim method

Assumptions underlying the ACPC method

- As there is no unique way of defining the Average Cost Per Claim (ACPC) method, there is no unique set of assumptions. In particular the assumptions relating to inflation will depend on the data used.
- In general terms, however, there are the assumptions that for each origin year, both the number and average amount of claims relating to each development year are constant proportions of the totals from that origin year.
- Finally, it is worth noting that for the assumptions to hold for this method, it would be normal for them to also hold for a simpler method applying to total rather than average claim amounts, such as the chain ladder method.

The average cost per claim method

Some valid questions

Question: Why do we not just use the basic chain ladder method?

Answer:

1. The totals used in the basic chain ladder method contain a combination of the patterns of the average amount and the numbers of claims.
2. By analysing these separately, we hope to get a more accurate projection.

Question: If an exam question asks for the Average Cost Per Claim method, what approach should I take?

Answer:

1. Unfortunately, you may need to use your judgement based on the form of the data given and the instructions given in the wording of the question.
2. It is possible that a variety of approaches may be equally acceptable - I am trying to test your understanding, not catch you out.

Loss ratios

Loss ratio = incurred claims / earned premium

- Investigation of the loss ratios for each of several different origin years would normally show some consistency, provided that there have not been any distortions and in particular no significant change in premium rates.
- The expected loss ratios will also have formed part of the derivation of the premium basis.
- It is therefore logical that a loss ratio based on trends of past data, underwriters' views, or market data, could be used as a basis for an estimate of the eventual loss and hence the outstanding claims. It is, however, on its own, a very crude measure due to the fluctuations that are inherent in any claims experience.
- The approach here is more general, in that the ultimate loss ratios can be estimated using any method, including subjective methods involving personal judgement.
- Once the ultimate estimated loss ratios have been found, they are applied to the premium figures in order to calculate outstanding claims reserve figures, just as we did before.

The Bornhuetter-Ferguson method

- The Bornhuetter-Ferguson method combines the estimated loss ratio with a projection method.
- It therefore improves on the crude use of a loss ratio by taking account of the information provided by the latest development pattern of the claims, while the addition of the loss ratio to a projection method serves to add some stability against distortions in the development pattern.

- The concepts behind the method are:

  1. That whatever claims have already developed in relation to a given origin year, the future development pattern will follow that experienced for other origin years.

  2. The past development for a given origin year does not necessarily provide a better clue to future claims than the more general loss ratio.

The Bornhuetter-Ferguson method

- In its simplest form the concept leads to the following approach to calculations:
  1. Determine the initial estimate of the total ultimate claims from each origin year using premiums and loss ratios.
  2. Divide these estimates by projection factors (f) determined, in a normal manner, from a claims development table. These are effectively estimates of the claims that should have developed to date.
  3. Subtract these amounts from the corresponding total ultimate claims figures to give an estimate of the amount of claims that are yet to develop.
- The 3 stages above can be combined as

$$\text{Future claims development} = \text{Premium} \times \text{Estimated Loss Ratio} \times (1 - 1/f)$$

- We can relate this formula to the three steps above as follows:

Step 1 gives you premium × expected loss ratio

Step 2 gives you premium × expected loss ratio × $1/f$

Step 3 gives you Step 1 minus Step 2, which is the formula given.

The Bornhuetter-Ferguson method

- In its original form, the Bornhuetter-Ferguson method was applied to the development of incurred claims. However, it could equally be applied to the development of paid claims, using either an accident year or policy year cohort.
- Further, the original projection was done using a chain-ladder approach, although alternative development factors or grossing-up factors (g) could easily be applied instead (*ie* g would replace $1/f$ in the expression for future claims development).
- The original form also made no explicit adjustment for inflation, although the method could be adjusted in a similar way to the other methods.
- The next example is based on the original form of the method, but you could also be asked to apply the method to paid claims.

The Bornhuetter-Ferguson method

Example

The first stage is to determine the development factors, using the same method as for the chain ladder methods.

	Development Year						
Accident Year	0	1	2	3	4	5	Ult
1	2,866	3,334	3,503	3,624	3,719	3,717	3,717
2	3,359	3,889	4,033	4,231	4,319		
3	3,848	4,503	4,779	4,946			
4	4,673	5,422	5,676				
5	5,369	6,142					
6	5,818						

RATIO (r)	1.158	1.049	1.039	1.023	0.999	1.000

This table shows cumulative incurred claims.

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The Bornhuetter-Ferguson method

Example

The "RATIO" figures are what we previously referred to as development factors. These are calculated in the usual way. E.g.: The ratio of DY1 to DY0 is given by:

(3,334 + 3,889 + 4,503 + 5,422 + 6,142) / (2,866 + 3,359 + 3,848 + 4,673 + 5,369)

= 23,290 / 20,115 = 1.158

Question: Confirm how the figure of 1.023 was calculated.

Answer:

- The ratio of DY4 to DY3 is given by:

(3,719 + 4,319) / (3,624 + 4,231) = 8, 038 / 7,855 = 1.023

The Bornhuetter-Ferguson method

Example

- Next, we find the earned premium (EP) for each accident year (AY). Earned premium are "new data" that would be derived from a separate source /provided in an exam.
- Then, the expected Ultimate Loss Ratio, say 83%, is applied to the earned premium to give the initial estimate of the ultimate loss (UL). The figure of 83% (or whatever) would normally be derived from a different source from the data in the triangle.

Accident Year (AY)	1	2	3	4	5	6
Earned Premium (EP)	4,486	5,024	5,680	6,590	7,482	8,502
Initial Ultimate Loss (0.83 EP)	3,723	4,170	4,714	5,470	6,210	7,057

- Note that in this example the expected Ultimate Loss Ratio has been taken as that experienced for the fully developed 1st accident year. This has been done purely on the grounds of lack of other information.

The Bornhuetter-Ferguson method

Example

- In other words the figure of 83% has been estimated by dividing the ultimate claims incurred for Accident Year 1 (3,717) by the earned premium for Accident Year 1 (4,486).
- If we knew that the claims experience was likely to be different for the other accident years, we would use different percentages for the other years.
- The next stage is the application of the development factors to the estimated ultimate losses and the addition of the incurred claims that have already been reported.
- Revised estimate of total ultimate losses by accident year:

AY	6	5	4	3	2	1
f	1.290	1.114	1.062	1.022	0.999	1.000
$1 - 1/f$	0.225	0.102	0.058	0.022	-0.001	0
Initial UL	7,057	6,210	5,470	4,714	4,170	3,723
Emerging liability	1,588	633	317	104	-4	0
Reported liability	5,818	6,142	5,676	4,946	4,319	3,717
Ultimate liability	7,406	6,775	5,993	5,050	4,315	3,717

The Bornhuetter-Ferguson method

Example

- *f* is cumulative development factor. E.g.: The figure of 1.290 is calculated as:

$$1.158 \times 1.049 \times 1.039 \times 1.023 \times 0.999 \times 1.000 = 1.290$$

- The emerging liability is calculated by multiplying the initial UL by the corresponding value of 1- 1/*f*

- The reported liability for a particular accident year is the last known figure in the run-off triangle for that accident year.
- The ultimate liability is the sum of the emerging liability and reported liability.
- The total ultimate liability relating to these six accident years is, therefore, 33,256.
- If the claims paid to date amounted to £12,256 (assumed number) then the total reserve required would be £21,000.
- The triangle is based on incurred claims so we cannot deduce total paid claims from it.