MTH5126 - Statistics for Insurance

Worksheet 7

Q1. Copula

(i) Write down Sklar's theorem.

(ii) Explain, in words, the meaning of the following copula expression: $C(u, v, w)$

The Gumbel copula has a generating function:
$$\psi(F(x)) = (-\ln F(x))^\alpha \text{ where } 1 \leq \alpha < \infty$$

(iii) Derive an expression for the Gumbel (Hougaard) copula for the case where there are three variables.

A student has fitted a Gumbel copula to investment returns from three developing markets, and has calculated a value for the dependency parameter, α, of 4.0.

She has separately determined that the probability of making a loss over the next calendar year (i.e., the probability that the return is less than 0%) in each of the three markets is 5%, 7.5% and 10% respectively.

(iv) Calculate the probability that all three markets have returns of less than 0% over the next calendar year.

(v) State what type of copula is equivalent to a Gumbel copula if $\alpha = 1.0$.

(vi) Calculate the probability that all three markets have returns of less than 0% over the next calendar year, assuming that each of the markets were independent.

Q2. Copula

Let X and Y be two random variables representing the future lifetimes of two 40-year old individuals. The two lives are married. You are given that:

$P(X \leq 20) = 0.17831$ and $P(Y \leq 20) = 0.11086$

Calculate the joint probability that both lives will die by the age of 60 using the Frank copula with $\alpha = 5$.

Q3. Copula

(i) Derive the coefficient of upper tail dependence for the Gumbel copula.
(ii) Comment on how the value of the parameter α affects the degree of upper tail dependence in the case of the Gumbel copula.