OVERVIEW OF MTH5104

- Supremum and infimum (least upper and greatest lower bounds). Completeness axiom.
- Sequences. Definition of convergence to 0. Demon game.
- Dominated convergence.
- Geometric sequences.
- Bounded sequences, convergent sequences are bounded, product of a sequence converging to 0 and a bounded sequence is a sequence converging to 0.
- Convergence of a sequence to $x \in \mathbb{R}$. Scaling a sequence by a constant; adding, multiplying and dividing two sequences.
- Suppose $(x_n)_{n=1}^{\infty}$ converges to x and $(y_n)_{n=1}^{\infty}$ converges to y, and that $x_n \leq y_n$ for all $n \in \mathbb{N}$. Then $x \leq y$.
- Monotonic sequences. Bounded monotonic sequences converge.
- Tending to ∞. An increasing sequence either converges or tends to ∞.
- Subsequences. Bolzano-Weierstrass: a bounded sequence contains a convergent subsequence.
- Accumulation point. Subsequence converging to $x \iff x$ is an accumulation point.
- If $(x_n)_{n=1}^{\infty}$ converges then $(x_{n+1} - x_n)_{n=1}^{\infty}$ converges to 0.
- Cauchy sequence. A sequence is Cauchy iff it is convergent.
- Series, definition of convergent series.
- If the *series* $\sum_{k=1}^{\infty} x_k$ converges, then the *sequence* $(x_k)_{k=1}^{\infty}$ converges to 0.
- Cauchy condition for convergence of a series.

- The sum of two convergent series is a convergent series. Scaling a convergent series by a constant yields a convergent series.

- Comparison test.

- Standard examples: Geometric series, harmonic series, alternating harmonic series, $\sum_{k=1}^{\infty} k^{-2}$.

- Absolutely convergent series. Absolute convergence implies convergence.

- The terms of an absolutely convergent series may be reordered without changing its value.

- A series that is convergent but not absolutely convergent can be reordered to achieve any desired value.

- Power series. A power series either (i) converges everywhere, (ii) converges only at $x = 0$, or (iii) converges for all $|x| < R$ and does not converge for all $|x| > R$ (R is the *radius of convergence*). NB: the case $|x| = R$ is undetermined.

- Definition of continuity for functions $\mathbb{R} \to \mathbb{R}$.

- Modified definition for functions defined on $D \subset \mathbb{R}$.

- Rules for combining continuous functions: scaling, adding, multiplying and dividing.

- The composition of continuous functions is continuous.

- Suppose $(x_n)_{n=1}^{\infty}$ is a sequence converging to a. If the function f is continuous at a then the sequence $(f(x_n))_{n=1}^{\infty}$ converges to $f(a)$.

- Definition of $\lim_{x\to a} f(x)$.

- Intermediate value theorem and its use in proving the existence of fixed points.

- Continuous functions on a closed interval. Suppose $f : [a, b] \to \mathbb{R}$ is continuous. Then:
  - f is bounded.
  - If $m = \inf\{f(x) : x \in [a, b]\}$ and $M = \sup\{f(x) : x \in [a, b]\}$ then there exist $c, d \in [a, b]$ with $f(c) = M$ and $f(d) = m$.
  - $f([a, b]) = [m, M]$.