3. HYPOTHESIS TESTS FOR TWO SAMPLES

3.1 TWO INDEPENDENT SAMPLES

		SAMPLE MEAN	SAMPLE VARIANCE
$X_1, \dots, X_{n_1}$	$\sim N(\mu_1, \sigma^2)$	$\bar{X}$	S_1^2
$Y_1, \dots, Y_{n_2}$	$\sim N(\mu_2, \sigma^2)$	$\bar{Y}$	S_2^2

$$\underbrace{\mu_1, \mu_2, \sigma^2}_{\text{POPULATION PARAMETERS}}$$

We want to test

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

Estimate a pooled variance:

$$S_0^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$\downarrow$

unbiased estimator of σ^2: that is, $E(S_0^2) = \sigma^2$

Also, it can be shown that

$$\frac{(n_1+n_2-2)S_0^2}{\sigma^2} \sim \chi^2_{n_1+n_2-2}$$

<u>IF σ^2 IS KNOWN</u>

$$\bar{X} \sim N\left(\mu_1, \frac{\sigma^2}{n_1}\right) \qquad \bar{Y} \sim N\left(\mu_2, \frac{\sigma^2}{n_2}\right)$$

$$\Rightarrow \quad \bar{X} - \bar{Y} \sim N\left((\mu_1 - \mu_2), \sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right)$$

We obtain a test statistic

$$Z = \frac{(\bar{X} - \bar{Y}) - \cancel{(\mu_1 - \mu_2)}}{\sigma\sqrt{1/n_1 + 1/n_2}} \sim N(0,1)$$

IF σ^2 IS NOT KNOWN

We replace σ^2 with S_0^2.

$$T = \frac{\bar{X} - \bar{Y}}{S_0\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1 + n_2 - 2}$$

3.2 F TEST FOR COMPARING TWO VARIANCES

$$\boxed{\text{thm: } \begin{array}{l} C_1 \sim \chi^2_{V_1} \\ C_2 \sim \chi^2_{V_2} \end{array} \Rightarrow \frac{C_1/V_1}{C_2/V_2} \sim F^{V_1}_{V_2}}$$

We know that

$$\frac{(n_1-1)S_1^2}{\sigma_1^2} \sim \chi^2_{n_1-1} \qquad \frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1}$$

and they are independent

$$\frac{\frac{(n_1-1)S_1^2}{\sigma_1^2} / (n_1-1)}{\frac{(n_2-1)S_2^2}{\sigma_2^2} / (n_2-1)} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{S_1^2/S_2^2}{\sigma_1^2/\sigma_2^2} \sim F^{n_1-1}_{n_2-1}$$

If we want to test

$$H_0: \sigma_1^2 = \sigma_2^2 \qquad H_1: \sigma_1^2 \neq \sigma_2^2$$

under H_0:

$$\frac{S_1^2}{S_2^2} \sim F^{n_1-1}_{n_2-1}$$ ← Test statistic

Example 3.6

$$H_0: \sigma_1^2 = \sigma_2^2 \quad \text{vs} \quad H_1 \quad \sigma_1^2 \neq \sigma_2^2$$

$$F = \frac{S_1^2}{S_2^2} = \frac{280.3}{310.3} = 0.9033$$

$$F \sim F^5_4$$

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

$F^{13-1=12}_{11-1=10}$

$F = \dfrac{93.3}{25.2} = \underline{\underline{3.70}} > 3.62$

$\Rightarrow$ reject null hypothesis

Example 3.9

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$T^* = \frac{\bar{X} - \bar{Y}}{\sqrt{S_1^2/n_1 + S_2^2/n_2}}$$