MTH6107 Chaos & Fractals

Exercises 5

EXAM QUESTION: the questions below correspond to the various parts of Question 3 on the January 2023 exam paper

For parameters $\lambda > 0$, define $f_\lambda : \mathbb{R} \to \mathbb{R}$ by

$$f_\lambda(x) = \lambda x^2(1-x)\,.$$

Exercise 1. Show that there is a point $p \in \mathbb{R}$ which is a fixed point of f_λ for all $\lambda > 0$. Is p attracting or repelling? Justify your answer.

Exercise 2. Determine the value $\lambda_1 > 0$ such that f_λ has precisely one fixed point if $\lambda \in (0, \lambda_1)$, and precisely 3 fixed points if $\lambda > \lambda_1$. Justify your answer.

Exercise 3. For $\lambda > \lambda_1$, let $x_\lambda^- < x_\lambda^+$ denote the two fixed points of f_λ which are not equal to p. Determine explicit formulae for x_λ^- and x_λ^+ in terms of λ.

Exercise 4. Show that x_λ^- is a repelling fixed point of f_λ for all $\lambda > \lambda_1$.

Exercise 5. Determine the value $\lambda_2 > \lambda_1$ such that if $\lambda \in (\lambda_1, \lambda_2)$ then x_λ^+ is an attracting fixed point of f_λ, and if $\lambda > \lambda_2$ then x_λ^+ is a repelling fixed point of f_λ. Justify your answer.

Exercise 6. Show that there exists $\lambda \in (5, 6)$ such that $2/3$ is a point of least period 2 for f_λ.