

# MTH6107 Chaos & Fractals

## Exercises 4

**EXAM QUESTION:** the questions below are based on the various parts of Question 2 on the January 2023 exam paper

Suppose  $a \geq 2$ , and that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x^2 - a$ .

**Exercise 1.** Determine all fixed points of  $f$ , and determine whether each fixed point is attracting or repelling, taking care to justify your answer.

**Exercise 2.** Determine all 2-cycles for  $f$ , and determine whether each 2-cycle is attracting or repelling, taking care to justify your answer.

**Exercise 3.** Give one example of an eventually fixed point that is not itself a fixed point, and one example of an eventually periodic point of least period 2 that is not itself a periodic point.

**Exercise 4.** If  $g : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $g(x) = x^2 + a$ , determine whether there is a topological conjugacy from  $f$  to  $g$ , taking care to justify your answer.

**Exercise 5.** If  $F : \mathbb{R} \rightarrow \mathbb{R}$  and  $G : \mathbb{R} \rightarrow \mathbb{R}$  are defined by  $F(x) = x - a$  and  $G(x) = x + a$ , determine whether there is a topological conjugacy from  $F$  to  $G$ , taking care to justify your answer.