PROBLEM SET 10 FOR MTH 6151

1. Use the Fourier-Poisson formula to compute the solution to the problem

$$U_t = \varkappa U_{xx}, \qquad x \in \mathbb{R}, \quad t > 0,$$
$$U(x,0) = 1.$$

Interpret the result you obtain. Is this surprising?

2. Use the Fourier-Poisson formula to find the limit as $t \to \infty$ of the solution to the problem

$$U_t = \varkappa U_{xx}, \qquad x \in \mathbb{R}, \quad t > 0,$$
$$U(x,0) = \begin{cases} 1 & -L < x < L \\ 0 & x < -L, \quad x > L \end{cases}.$$

Plot the solutions at several instants of time and describe in qualitative terms the behaviour of the solution to as $t \to \infty$. What is $\lim_{t\to\infty} U(x,t)$?

3. Use the Fourier-Poisson formula to find the limit as $t \to \infty$ of the solution to the problem

$$U_t = \varkappa U_{xx}, \qquad x \in \mathbb{R}, \quad t > 0,$$
$$U(x,0) = \begin{cases} 3 & x < 0 \\ 1 & x > 0 \end{cases}.$$

Plot the solutions at several instants of time and describe in qualitative terms the behaviour of the solution to as $t \to \infty$. What is $\lim_{t\to\infty} U(x,t)$?

4. Use the Fourier-Poisson formula to compute the solution to the problem

$$U_t = \varkappa U_{xx}, \qquad x \in \mathbb{R}, \quad t > 0,$$
$$U(x,0) = e^{3x}.$$

5. Use the Fourier-Poisson formula to compute the solution to the problem

$$U_t = \varkappa U_{xx}, \qquad x \in \mathbb{R}, \quad t > 0,$$
$$U(x,0) = \begin{cases} 0 & x < 0 \\ e^{-x} & x > 0 \end{cases}.$$

What happens as $t \to \infty$.

6. Use even extensions to find the solution to the problem on the half-line with Neumann boundary conditions

$$U_t = \varkappa U_{xx}, \qquad x > 0, \quad t > 0,$$
$$U(x,0) = f(x),$$
$$U_x(0,t) = 0.$$

7. Consider the solution

$$U(x,t) = x^2 + 2\varkappa t$$

of the heat equation. Find the location of its maximum and minimum in the rectangle

$$\{0 \leq x \leq 1,\ 0 \leq t \leq T\}.$$