QUIZ ANSWERS

OA 30th 2022

Q1 On $\mathbb{R}$

			Roots multiplicity	1st	2nd	3rd	4th
(a)	$\dot{x} = x^4(x-2)(x-3)(x-4)$	✗	x^7	4	1	1	1
(b)	$\dot{x} = x^4(x-1)(x-2)(3-x)$	✓	$-x^7$	4	1	1	1
(c)	$\dot{x} = x(x-1)(x-2)(x-3)$	✗	x^4	1	1	1	1
(d)	$\dot{x} = -x^2(x-1)(x-2)(x-3)$	✓	$-x^5$	2	1	1	1
(e)	$\dot{x} = -x^2(x-2)(x-3)(x-4)$	✓	$-x^5$	2	1	1	1

Q2 On $\mathbb{R}$ $\dot{x} = x^2$

- ✗ both stable and unstable
- ✗ asymptotically stable
- ✓ unstable
- ✗ stable

Q3 $\dot{\theta} = \sin^2(\theta)$, fixed pt $\theta = 0$

✗ asymptotically stable

✓ has a non-trivial basin of attraction (S^1)

✗ stable

✓ unstable

✗ neither stable nor unstable

Q4

✗ $\dot{x} = x(x-1)^2(x-2)(x-3) \sim +x^5$ 1 2 1 1

✓ $\dot{x} = -x^2(x-2)(x-3)(x-4)^3 \sim -x^7$ 2 1 1 3

✓ $\dot{x} = x^2(x-1)(x-2)^3(3-x) \sim -x^7$ 2 1 3 1

✓ $\dot{x} = -x^4(x-1)(x-2)(x-3) \sim -x^7$ 4 1 1 1

✗ $\dot{x} = x^4(x-2)(x-3)(x-4) \sim +x^7$ 4 1 1 1

Q5

- asymptotically stable is also stable (F)
- stable fixed point of a linear system is always asymptotically stable (on the plane) (F)
- a basin of a linear system can be one dimensional (T) (on line and plane)
- a fixed point of a linear system on the plane is asymptotically stable if all orbits converge asymptotically to the fixed point (T)
- a fixed point of a system on the circle is asymptotically stable if all orbits asymptotically converge to the fixed point (F)

Q6

$\dot{x} = y$, $\dot{y} = -4x$ — stable, but not AS

$\dot{x} = -2$, $\dot{y} = -3y$ — AS (∴ also stable)

$\dot{x} = 0$, $\dot{y} = 0$ — stable, but not AS

$\dot{x} = 0$, $\dot{y} = y$ — unstable

$\dot{\theta} = 1 - \cos\theta$, unstable, unstable with $B(\theta=0) = S^1$

$\dot{\theta} = \sin^2(2\theta)$, unstable, "none of these"
not stable, not $B(\theta=0) = S^1$