Which of the following ODEs on $\mathbb{R}$ give a phase portrait which is qualitatively the same as the one illustrated below?

Select one or more:

- ☐ $\dot{x} = x^4(x-2)(x-3)(x-4)$
- ☐ $\dot{x} = +x^4(x-1)(x-2)(3-x)$
- ☐ $\dot{x} = x(x-1)(x-2)(x-3)$
- ☐ None of these.
- ☐ $\dot{x} = -x^2(x-1)(x-2)(x-3)$
- ☐ $\dot{x} = -x^2(x-2)(x-3)(x-4)$

The fixed point at $x = 0$ of the ODE $\dot{x} = x^4$ is

Select one:

- ○ both stable and unstable
- ○ none of these
- ○ asymptotically stable
- ○ unstable
- ○ stable

Consider the ODE on the circle $\mathbb{S}$ given by $\dot{\theta} = \sin^2(\theta)$. The fixed point at $\theta = 0$

Select one or more:

- ☐ a. asymptotically stable
- ☐ b. has a non-trivial basin of attraction
- ☐ c. stable
- ☐ d. unstable
- ☐ e. neither stable nor unstable
- ☐ f. None of these.

Which of the following ODEs on $\mathbb{R}$ give a phase portrait which is qualitatively the same as the one illustrated above?

Select one or more:

- ☐ $\dot{x} = (1+x)x^2(x+2)^2(x+4)$
- ☐ $\dot{x} = x(x-1)^2(x-2)(x-5)^2$
- ☐ $\dot{x} = x(x-1)(x-2)(x-3)$
- ☐ $\dot{x} = (x+2)(x+1)^2(x-1)(x-2)^2$
- ☐ $\dot{x} = -(x+1)(x+2)(x+3)^2(x+4)^2$
- ☐ None of the other answers.

Which of the following ODEs on $\mathbb{R}$ give a phase portrait which is qualitatively the same as the one illustrated below?

Select one or more:

- ☐ $\dot{x} = x(x-1)^2(x-2)(x-3)$
- ☐ $\dot{x} = -x^2(x-2)(x-3)(x-4)^3$
- ☐ $\dot{x} = +x^2(x-1)(x-2)^3(3-x)$
- ☐ $\dot{x} = -x^4(x-1)(x-2)(x-3)$
- ☐ None of these.
- ☐ $\dot{x} = x^4(x-2)(x-3)(x-4)$

Identify as to whether the following statements are true or false:

An asymptotically stable fixed point of a linear system on the plane is also stable [ ]

A stable fixed point of a linear system is always asymptotically stable - [ ]

A basin of attraction of a fixed point of a linear system can be one dimensional - [ ]

A fixed point of a linear system on the plane is asymptotically stable if all orbits asymptotically converge to the fixed point as $t \to \infty$ [ ]

A fixed point of a system on the circle is asymptotically stable if all orbits asymptotically converge to the fixed point as $t \to \infty$ [ ]

See below for choices!

Choice A =
- unstable
- asymptotically stable
- none of these
- stable fixed point, but not asymptotically stable

Choice B
- unstable with basin of attraction the whole circle
- none of these
- stable

Remind yourself of the definitions of

- unstable fixed point
- stable fixed point, but not asymptotically stable
- asymptotically stable fixed point

Which is the type of fixed point at the origin for each of the following systems on $\mathbb{R}^2$

$\dot{x} = y, \dot{y} = -4x$ - Choice A

$\dot{x} = -2x, \dot{y} = -3y$ - Choice A

$\dot{x} = 0, \dot{y} = 0$ - Choice A

$\dot{x} = 0, \dot{y} = y$ - Choice A

and on the circle $\mathbb{S}$ at the fixed point $\theta = 0$ for each of

$\dot{\theta} = 1 - \cos(\theta)$ - Choice A Choice B

$\dot{\theta} = \sin^2(2\theta)$ - Choice A Choice B.