# QUEEN MARY, UNIVERSITY OF LONDON

## MTH6102: Bayesian Statistical Methods

### Exercise sheet 5

### 2023-2024

1. If the data follow a normal distribution with unknown mean $\mu$ and known standard deviation, and $\mu$ is assigned a normal prior distribution, show that the posterior mean for $\mu$ can be written as a weighted sum of the maximum likelihood estimate and the prior mean.

2. The column in the exercise sheet 5 dataset, labelled `x`, contains the observed data data be $x_1, \ldots, x_n$. Suppose that each data-point $x_i$ is normally distributed, with unknown mean $\theta$ and standard deviation assumed to be equal to 1.

   As a prior distribution for $\theta$, we assign a normal distribution with mean 5 and standard deviation 10.

   (a) What is the posterior distribution for $\theta$?

   (b) What is the posterior mean for $\theta$?

   (c) What is the posterior median for $\theta$?

   (d) What is a 95% equal tail credible interval for $\theta$?

3. Let $x_1, \ldots, x_n$ be iid Poisson($\lambda$), and let $\lambda$ have a gamma($\alpha, \beta$) distribution.

   (a) Show that gamma($\alpha, \beta$) is conjugate to the Poisson likelihood.

   (b) Calculate the posterior mean and variance.

   (c) Show how to find a 95% equal tail credible interval for $\lambda$?

   (d) Show how to find a 95% HPD credible interval for $\lambda$.

4. In an investigation into the size of the errors produced by a new measurement instrument, $n$ measurements are taken of a standard sample of mass 1000 grams. The measurements (in grams), $y_1, \ldots, y_n$ can be modelled as a random sample from a normal distribution with known mean $\mu = 1000$ and unknown precision $\tau$ (reciprocal of the variance). Assume the prior gamma($\alpha$, $\beta$) distribution on $\tau$, where $\alpha = 5$ and $\beta = 0.05$

   (a) Six measurements are taken and the data is

   $$1000.11, 999.96, 999.84, 999.89, 999.80, 1000.09.$$

   Given these measurements, what is the posterior distribution of $\tau$?

   (b) Use R to find the posterior median and a 95% equal tail credible interval for $\tau$.

   (c) Can you find the posterior median for $\sigma = 1/\sqrt{\tau}$? Can you find a 95% credible interval for $\sigma$? [Hint: these do not need the derivation of the posterior distribution for $\sigma$, or any extensive calculations.]