Developing the distribution function of (D_i, V_i)

$f_i(d_i, v_i)$ is the joint p.d.f. of (D_i, V_i) where

if $d_i = 0$ $\quad f_i(d_i, v_i) = {}_{b_i - a_i}p_{x+a_i}$ $\quad$ survives

if $d_i = 1$ $\quad f_i(d_i, v_i) = {}_{v_i}p_{x+a_i}\,\mu_{x+a_i+v_i}$ $\quad$ dies

then using ${}_tp_x = \exp\left(-\int_0^t \mu_{x+s}\,ds\right)$ gives

$$f_i(d_i, v_i) = \begin{cases} \exp\left(-\int_0^{b_i-a_i} \mu_{x+a_i+t}\,dt\right) & \text{if } d_i = 0 \\ \exp\left(-\int_0^{v_i} \mu_{x+a_i+t}\,dt\right)\mu_{x+a_i+v_i} & \text{if } d_i = 1 \end{cases}$$

$$= \exp\left(-\int_0^{v_i} \mu_{x+a_i+t}\,dt\right)\mu_{x+a_i+v_i}^{d_i}$$

introducing power of d_i (0 or 1) allows one common formula

if the transition intensity μ_{x+t} is constant μ for values of t we observing then the integral does not vary with t and $\int_0^{v_i} \mu_{x+a_i+t}\,dt = \mu v_i$

then with a constant μ

$$f_i(d_i, v_i) = \exp(-\mu v_i) \cdot \mu^{d_i}$$

then the joint probability function $g(\partial_i, V_i)$ for our N observations is

$$\prod_{i=1}^{N} f_i(d_i, v_i) = \prod_{i=1}^{N} \exp(-\mu v_i)\, \mu^{d_i}$$

$$= \exp[-\mu(v_1 + v_2 + \dots + v_N)] \cdot \mu^{d_1 + d_2 + \dots + d_N}$$

$$= \underline{\underline{\exp(-\mu v) \cdot \mu^d}}$$ where

$d = \sum_{i=1}^{N} d_i$ total nº of deaths

$v = \sum_{i=1}^{N} v_i$ total waiting time