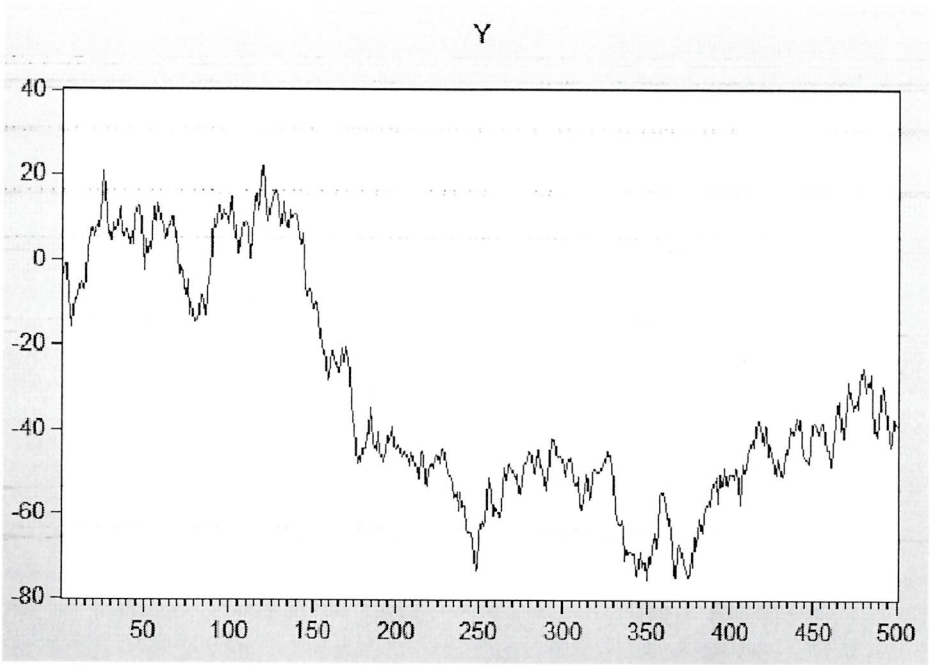
Examples. Testing for unit root

Model 1. Simulated data from random walk:

$$Y_t = Y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$



Augmented Dickey-Fuller test:
– constant and linear trend,
– Schwarz Information Criteria (SIC or BIC) for choosing augmentation lag, maximum lag 10

$$\Delta Y_t = c + bt + b_c Y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta Y_{t-j} + \varepsilon_t$$

$$\Delta Y_t = Y_t - Y_{t-1}$$

$p-1=0$ No augmentation

Model

$$Y_t = Y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

Augmented Dickey-Fuller Unit Root Test on Y

Null Hypothesis: Y has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

Estimated

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.647438	0.7727
Test critical values:	1% level	-3.976554	
	5% level	-3.418852	
	10% level	-3.131965	

$0.7727 > 0.05$

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 12/11/20 Time: 20:32
Sample (adjusted): 2 500
Included observations: 499 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
β_c = Y(-1)	-0.012634	0.007669	-1.647438	0.1001
C	-0.193345	0.284205	-0.680303	0.4966
@TREND("1")	-0.001167	0.001429	-0.816862	0.4144

Not sign. Not sign

R-squared	0.006038	Mean dependent var	-0.072537
Adjusted R-squared	0.002030	S.D. dependent var	3.171401
S.E. of regression	3.168180	Akaike info criterion	5.150185
Sum squared resid	4978.533	Schwarz criterion	5.175512
Log likelihood	-1281.971	Hannan-Quinn criter.	5.160124
F-statistic	1.506553	Durbin-Watson stat	1.934103
Prob(F-statistic)	0.222685		

$\beta_c = 1 - \beta$; $\beta = 1 + \beta_c =$
$= 1 - 0.012 =$
$= 0.988$

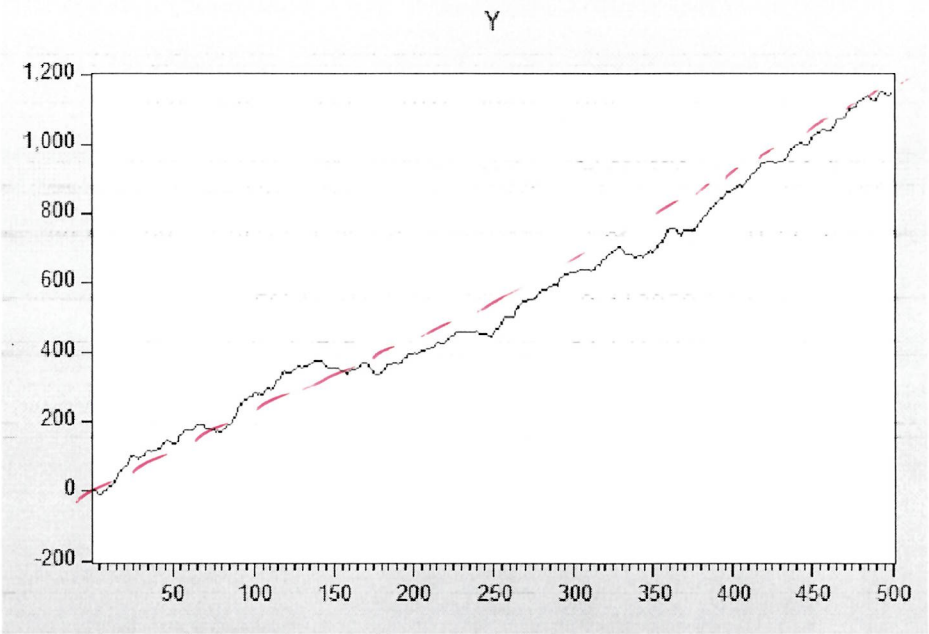
Fitted model

$$Y_t = Y_{t-1} + \varepsilon_t$$

Model 2. Simulated data from random walk with drift:

$$Y_t = 1 + Y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

ε_t white noise



Augmented Dickey-Fuller test:
– constant and linear trend,
– Schwarz Information Criteria (SIC or BIC) for choosing augmentation lag, maximum lag 10

BIC is used to select the number of lags in augmentation part

$$\sum_{j=1}^{p-1} \phi_j \Delta Y_{t-j}$$

"number of lags" = $p-1$

$$\Delta Y_t = c + bt + \beta_c Y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta Y_{t-j} + \varepsilon_j$$

white noise

Model need to generate data

$$Y_t = 1 + 1.6Y_{t-1} - 0.6Y_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

Augmented Dickey-Fuller Unit Root Test on Y

Null Hypothesis: Y has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

$p-1 =$ No augmentation needed

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.647438	0.7727
Test critical values:	1% level	-3.976554	
	5% level	-3.418852	
	10% level	-3.131965	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 12/11/20 Time: 20:42
Sample (adjusted): 2 500
Included observations: 499 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
β_c Y(-1)	-0.012634	0.007669	-1.647438	0.1001
C	0.806655	0.284205	2.838287	0.0047
b = @TREND("1")	0.011467	0.006706	1.709873	0.0879

signific. / signific / not signif.

R-squared	0.006038	Mean dependent var	0.927463
Adjusted R-squared	0.002030	S.D. dependent var	3.171401
S.E. of regression	3.168180	Akaike info criterion	5.150185
Sum squared resid	4978.533	Schwarz criterion	5.175512
Log likelihood	-1281.971	Hannan-Quinn criter.	5.160124
F-statistic	1.506553	Durbin-Watson stat	1.934103
Prob(F-statistic)	0.222685		

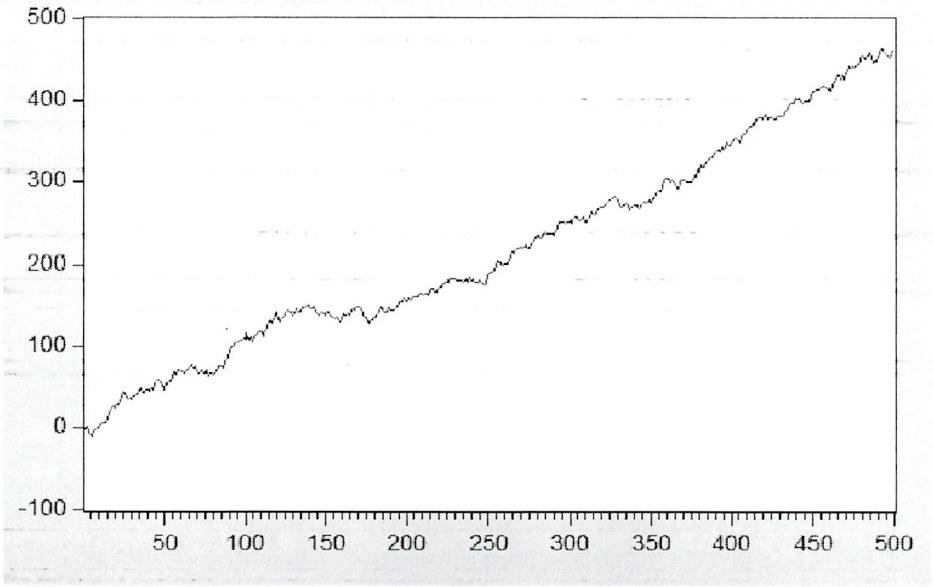
FITTED model

$$Y_t = 0.8 + Y_{t-1} + \varepsilon_t$$

Model 3. Simulated data from random walk with drift:

$$Y_t = 1 + 1.6Y_{t-1} - 0.6Y_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

$$= 1 + Y_{t-1} + 0.6(Y_t - Y_{t-1}) + \varepsilon_t$$



Augmented Dickey-Fuller test:
– constant and linear trend,
– Schwarz Information Criteria (SIC or BIC) for choosing augmentation lag, maximum lag 10

$$\Delta Y_t = c + bt + \beta_c Y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta Y_{t-j} + \varepsilon_j$$

Model

$$Y_t = 1 + 1.6Y_{t-1} - 0.6Y_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

Augmented Dickey-Fuller Unit Root Test on Y

Null Hypothesis: Y has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=10)

$p-1 = 1$

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.531377	0.8177
Test critical values:	1% level	-3.976591	
	5% level	-3.418870	
	10% level	-3.131976	

*MacKinnon (1996) one-sided p-values.

Do not reject unit root

$H_0 : \beta_c = 0 \iff \beta = 1$

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 12/11/20 Time: 20:47
Sample (adjusted): 3 500
Included observations: 498 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.	
β_c = Y(-1)	-0.004741	0.003096	-1.531377	0.1263	Not signif.
ϕ_1 = D(Y(-1))	0.585177	0.036600	15.98833	0.0000	Signif.
C	0.837509	0.295628	2.832983	0.0048	Signif
@TREND("1")	0.010762	0.006683	1.610371	0.1080	Not

R-squared	0.343142	Mean dependent var	2.290371
Adjusted R-squared	0.339153	S.D. dependent var	3.906970
S.E. of regression	3.176073	Akaike info criterion	5.157168
Sum squared resid	4983.195	Schwarz criterion	5.190988
Log likelihood	-1280.135	Hannan-Quinn criter.	5.170441
F-statistic	86.02178	Durbin-Watson stat	1.923549
Prob(F-statistic)	0.000000		

FITTED Model

$$Y_t = 0.83 + Y_{t-1} + 0.58\Delta Y_{t-1} + \varepsilon_t$$

$$= 0.83 + Y_{t-1} + 0.58(Y_{t-1} - Y_{t-2}) + \varepsilon_t$$

$$= 0.83 + 1.58Y_{t-1} - 0.58Y_{t-2} + \varepsilon_t$$