Questions Week 2 – Solutions

1. the central rate of mortality m_x is

$m_x = q_x / \int_0^1 {}_tp_x \, dt$

now we also know that $q_x = \int_0^1 {}_tp_x \, \mu_{x+t} \, dt$ and if the force of mortality is a constant μ then

$q_x = \int_0^1 {}_tp_x \, \mu \, dt = \mu \int_0^1 {}_tp_x \, dt$ and then

$m_x = q_x / \int_0^1 {}_tp_x \, dt = \mu \int_0^1 {}_tp_x \, dt / \int_0^1 {}_tp_x \, dt = \mu$

2. we seek the discrete expectation of life (in days) which is an e_x type measure

and $e_x = \sum_{all\ k} {}_kp_x$

The question gives a set of ${}_1q$ type probabilities

Day (k)	1	2	3	4	5
${}_1q$	0.25	0.46	0.64	0.85	1
${}_1p$	0.75	0.54	0.36	0.15	0
${}_kp$	0.75	0.405	0.1458	0.02187	0

so " e_x " = 0.75 + 0.405 + 0.1458 + 0.02187 + 0 = 1.32267 days

3. (a) the force of mortality is $\mu_x = B\,c^x$ where B, c are constants

(b) we are given

$\mu_{45} = B\,c^{45} = 0.0046$

$\mu_{46} = B\,c^{46} = 0.0048$

therefore dividing these 2 equations $c^{46} / c^{45} = 48/46 = 24/23$

therefore

c = 24/23 = 1.043487

B = 0.000678

4. under the exponential model S(t) = exp(-μt) where we measure t in days here

from the first day we estimate μ = 24/64000 = 0.000375

probability of still working after a year is S(365) = exp(-0.000375 x 365) = 0.872

5. we know that $m_{70} = q_{70} / \int_0^1 {}_tp_{70}\, dt$

now $\int_0^1 {}_tp_x\, dt \le 1$ and in fact is < 1 unless ${}_tp_x = 1$ all t (which in practice is never the case)

therefore, m_{70} is larger than q_{70}