Lecture 3B
MTH6102: Bayesian Statistical Methods

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Today's agenda

Today's lecture

- Conjugate priors for the normal likelihood
- Update a gamma prior for the inverse of the normal variance given a normal likelihood with known mean and unknown variance.

Normal example, known variance

- $y_1, \dots, y_n \sim N(\mu, \sigma^2)$.
- We saw that a normal prior distribution for μ is conjugate in this case.
- It's conjugate because it results in a posterior in the same family as the prior.

Normal example, known variance

- Observed data $y_1, \ldots, y_n \sim N(\mu, \sigma^2)$.
- Prior distribution $\mu \sim N(\mu_0, \sigma_0^2)$.
- The posterior distribution is

$$\mu \sim N(\mu_1, \sigma_1^2)$$

where

$$\mu_1 = \left(\frac{\mu_0}{\sigma_0^2} + \frac{n\bar{y}}{\sigma^2}\right) \Big/ \left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}\right)$$

$$\sigma_1^2 = 1 \Big/ \left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}\right)$$

Normal example, known mean

- Observed data $y_1, \ldots, y_n \sim N(\mu, \sigma^2)$.
- Suppose that μ is known and σ is unknown.
- It is easier to work with $\tau = 1/\sigma^2$.
- τ is known as the precision.
- τ is the reciprocal of the variance, so large τ means small variance and hence high precision.

Normal example, known mean

- In this case, a gamma prior is conjugate for τ.
- Prior $\tau \sim \text{Gamma}(\alpha, \beta)$.
- Posterior

$$\tau \sim \text{Gamma}\left(\alpha + \frac{n}{2}, \beta + \frac{\sum_{i=1}^{n}(y_i - \mu)^2}{2}\right)$$

- The family of gamma distributions is conjugate to the normal likelihood for the normal precision parameter τ, if the mean μ is known.

Normal example, both parameters unknown

- If μ and τ are unknown then there is a bivariate distribution which is conjugate.
- Marginal distribution

$$\tau \sim \text{Gamma}$$

  and conditional distribution

$$\mu \mid \tau \sim \text{Normal.}$$

- The joint prior distribution is the product of these two.
- The posterior is of the same form.
- We're not going into details in this module.