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MTH6102: Bayesian Statistical Methods

Exercise sheet 3

2023-2024

1. Continue question 2 of exercise sheet 1, about the lifetimes of mosquitoes.

   Suppose we take a $\text{Gamma}(0.1, 2)$ distribution as a prior distribution for λ. With observed lifetimes $(4, 12, 6, 19, 9, 5)$, what is the posterior distribution for λ?

   Now suppose that the data was as above, except that instead of dying after 19 days, the fourth mosquito was observed to be still alive after 20 days, with the exact time of death not known. What is the posterior distribution for λ?

2. Let the observed data be $y = (y_1, \ldots, y_n)$, a random sample from the Poisson distribution with mean λ, where λ is unknown.

   Write down the likelihood for this problem. Find an expression for $\hat{\lambda}$, the maximum likelihood estimate (MLE) for λ.

   Suppose that $y = (6, 4, 9, 2, 0, 3)$. What is $\hat{\lambda}$ for this dataset?

   Show that with a $\text{Gamma}(\alpha, \beta)$ prior distribution for λ, the posterior distribution is also gamma-distributed with parameters that you should determine. With prior parameters $\alpha = 1, \beta = 1$, what is the posterior distribution for λ for the above data?

3. Consider the `iris` dataset in R that we saw in practical 2. Let $y_1, \ldots, y_n$ be the column `iris$Sepal.Length`, and assume that they are independent. Suppose that each $y_i \sim N(\mu, \sigma^2)$, where we assume σ is known and equal to 0.9. For μ, we assign a normal prior distribution $N(\mu_0, \sigma_0^2)$, where $\mu_0 = 5$, $\sigma_0 = 2$. Find the posterior distribution of μ, after seeing the `iris` dataset.

4. Suppose now that in question 3, we take μ as known, equal to 5, and let $\tau = 1/\sigma^2$. For τ, we assign a prior distribution that is $\text{Gamma}(2, 2)$. Find the posterior distribution of τ, after seeing the `iris` dataset.