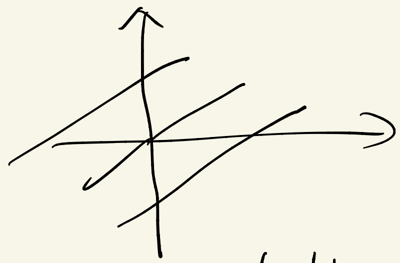
Selected solutions to Problem Set 2 (continued)

2 (2) This is an initial value problem (IVP)

step 1. The characteristic lines are given by

$bx - ay = c$ with $a = \pi$, $b = 1$

i.e. $x - \pi y = c$ (characteristic line)



so the general solution is

$u(x, y) = f(c) = f(x - \pi y)$, for any function f.
(general solution)

step 2. Next we specify f using the initial value

$f(c) = u(x, 0) = x^2$ when $y = 0$

on the on the other hand,
when $y = 0$, we get from the characteristic line equation that $x = c$

so $f(c) = u(x,0) = x^2 = c^2$

plug into the expression of f to the general solution, we get

$u(x,y) = (x-\pi y)^2$

3. (1) $\frac{\partial}{\partial x}(x+e^t) = 1$

$\frac{\partial}{\partial t}(x+e^t) = e^t$

and $\frac{\partial}{\partial x}(x+e^t) + \frac{\partial}{\partial t}(x+e^t) = 1+e^t$

so it is a solution.

(2). Step 1: solve the general solution for the homogeneous equation

$\tilde{u}_x + \tilde{u}_t = 0$,

get $\tilde{u}(x,t) = f(x-t)$, for any f

step 2: By the principle of superposition, and combining step 1, the general solution to the inhomogeneous equation is

$u(x,t) = 1 + e^t + f(x-t)$, for any f