"important formula" no 2 : ${}_tp_x =$

we begin by considering $\frac{d}{ds}\,{}_sp_x$

$$\frac{d}{ds}\,{}_sp_x = \frac{d}{ds}(1 - {}_sq_x) = -\frac{d}{ds}\,{}_sq_x = -f_x(s)$$

$$= -\,{}_sp_x\,\mu_{x+s}$$

now seperately, we consider $\frac{d}{ds}\log {}_sp_x$

Recall from Calculus that

$$\frac{d}{dy}\log y = \frac{1}{y} \quad \text{and} \quad \frac{d}{dy}\log h(y) = \frac{\frac{d}{dy}h(y)}{h(y)}$$

so $\frac{d}{ds}\log {}_sp_x = \dfrac{\frac{d}{ds}\,{}_sp_x}{{}_sp_x}$

substituting in our earlier result for $\frac{d}{ds}\,{}_sp_x$ gives:

$$\frac{d}{ds}\log {}_sp_x = -\frac{{}_sp_x\,\mu_{x+s}}{{}_sp_x}$$

$$\therefore \frac{d}{ds}\log {}_sp_x = -\mu_{x+s}$$

Now integrate both sides over $s = (0, t)$

$$\frac{d}{ds} \log {}_sp_x = -\mu_{x+s}$$

$$\int_0^t \frac{d}{ds} \log {}_sp_x \, ds = -\int_0^t \mu_{x+s} \, ds$$

$$\Big[\log {}_sp_x \Big]_0^t = -\int_0^t \mu_{x+s} \, ds$$

$\log {}_tp_x - \log {}_0p_x$ and as ${}_0p_x = 1$

$\log {}_0p_x = 0$

$$\log {}_tp_x = -\int_0^t \mu_{x+s} \, ds$$

$$\underline{\underline{{}_tp_x = e^{-\int_0^t \mu_{x+s} \, ds}}}$$