Curtate expectation of life e_x

$$e_x = \sum_{k=0}^{\omega-x} k \cdot {}_kp_x \, q_{x+k}$$

$$= 1 \times {}_1p_x \times q_{x+1} + 2\,{}_2p_x\, q_{x+2} + 3\,{}_3p_x\, q_{x+3} + \ldots.$$

$$= {}_1p_x\, q_{x+1}$$

$$+ {}_2p_x\, q_{x+2} + {}_2p_x\, q_{x+2}$$

$$+ {}_3p_x\, q_{x+3} + {}_3p_x\, q_{x+3} + {}_3p_x\, q_{x+3}$$

$$+ \ldots.$$

now sum columns

live for 2 years then die in next year
+ live for 3 years then die in next year
+ ... keep summing to limiting age

this represents all deaths after $x+2$

which in turn represents all that survive from x to $x+2$
$= {}_2p_x$

and summing all the columns

$$e_x = {}_1p_x + {}_2p_x + {}_3p_x + \ldots.$$

$$e_x = \sum_{k=1}^{\omega-x} {}_kp_x$$