Expected future lifetime $\mathring{e}_x$

$\mathring{e}_x$ is $E[T_x]$ and in general

$$E[T_x] = \int_0^\infty t\, f_x(t)\, dt$$

now $f_x(t) = \dfrac{d}{dt} F_x(t) = \dfrac{d}{dt}\, {}_tq_x = \dfrac{d}{dt}(1 - {}_tp_x)$

$$\therefore\ E[T_x] = \int_0^{\omega - x} t . f_x(t)\, dt = \int_0^{\omega - x} t . \frac{d}{dt}(1 - {}_tp_x)\, dt$$

$$= \int_0^{\omega - x} t . \left(-\frac{d}{dt}\, {}_tp_x \right) dt$$

we can now integrate by parts

Recall from Calculus : "Integration by parts"

$$\int g(x) \frac{d}{dx} h(x)\, dx = g(x)\, h(x) - \int h(x) \frac{d}{dx} g(x)\, dx$$

$$E(T_x) = -\left[t .\, {}_tp_x \right]_0^{\omega - x} + \int_0^{\omega - x} {}_tp_x\, dt$$

this is zero when evaluated at $t=0$ and at $t = \omega - x$

$$\therefore\ \mathring{e}_x = E(T_x) = \int_0^{\omega - x} {}_tp_x\, dt$$