MTH6142 Complex Networks

Assessed Coursework 5

Consider the following growing network model in which each node i is assigned an *attractiveness* $a_i \in \mathbb{N}^+$ drawn from a distribution $\pi(a)$.
Let $N(t)$ denote the total number of nodes at time t.
At time $t = 1$ the network is formed by two nodes joined by a link.

- At every time step a new node joins the network. Every new node has initially a single link that connects it to the rest of the network.
- At every time step t the link of the new node is attached to an existing node i of the network chosen with probability Π_i given by

$$\Pi_i = \frac{a_i}{Z},$$

where

$$Z = \sum_{j=1,\ldots,N(t-1)} a_j.$$

Provide the mean-field solution of the model by considering the following two points.

(A) Assume that

$$Z \simeq \overline{a}t,$$

where $\overline{a}$ indicates the average of a over the distribution $\pi(a)$.
Derive the time evolution $k_i = k_i(t)$ of the expected degree k_i of a node i in the mean-field approximation. **[2 MARKS]**

(B) Assume that

$$\pi(a) = \begin{cases} 1 & \text{for} \quad a = 1, \\ 0 & \text{for} \quad a \neq 1, \end{cases}$$

and that $Z \simeq \overline{a}t$.
Derive the degree distribution $P(k)$ of the network for large times, i.e. $t \gg 1$, in the mean-field approximation. **[2 MARKS]**