Queen Mary
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# Complex Networks (MTH6142)
# Formative Assignment 5

- **1. Random networks in the $\mathbb{G}(N,p)$ ensemble**
  Assume that $p = a/N^z$, where $a > 0$ and $z \geq 0$, and $a, z$ independent on $N$.

  (a) Determine the average degree $\langle k \rangle$ in the limit $N \to \infty$ for the following values of the parameters
    (i) $a = 0.5, z = 1$;
    (ii) $a = 2, z = 1$;
    (iii) $a > 0,\ z = 2$;
    (iv) $a > 0, z = 0.5$.

  (b) In which of the above cases does the random network contain a giant component in the limit $N \to \infty$?.

  (c) Given $p = a/N^z$ with generic values of $a > 0, z \geq 0$ determine the average degree $\langle k \rangle$ in the large network limit $N \to \infty$.

  (d) Determine the conditions on $a$ and $z$ for these random networks to be subcritical, i.e. with a fraction $S$ of nodes in the giant component given by $S = 0$ in the $N \to \infty$ limit.

  (e) Determine the conditions on $a$ and $z$ for these random networks to be supercritical, i.e. with a non vanishing fraction $S$ of nodes in the giant component ($S > 0$) in the $N \to \infty$ limit.

  (f) Determine the conditions on $a$ and $z$ for which these random networks are critical, in the large network limit, i.e. in the limit $N \to \infty$.

- **2. Random networks in the $\mathbb{G}(N,p)$ ensemble with $p = c/(N-1)$ where $c > 0$.**

  (a) Calculate the average number of triangles $\mathcal{N}_3^{\text{triangles}}$ in the network, by evaluating first the number of ways to select 3 nodes out of $N$ nodes, and secondly the probability that the selected nodes are all connected to each other.

  (b) Show that in the limit $N \to \infty$ the average number of triangles in the network is

$$\mathcal{N}_3^{\text{triangles}} \simeq \frac{1}{6}c^3. \tag{1}$$

  This means that the number of triangles is constant, neither growing or vanishing, in the limit of large $N$.