Complex Networks (MTH6142)
Formative Assignment 3

- **1. Centrality measures of a given directed network**
  Consider the adjacency matrix $\mathbf{A}$ of a directed network of size $N = 4$ given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In the following we will indicate with $\mathbf{1}$ the column vector $\mathbf{1}$ with elements $1_i = 1\ \forall i = 1, 2 \ldots, N$ and we will indicate with $\mathbf{I}$ the identity matrix.

(a) Draw the network

(b) Calculate the eigenvector centrality using its definition.
Is the result expected? (*Explain why*).

(c) Calculate the Katz centrality

$$\mathbf{x} = \beta(\mathbf{I} - \alpha\mathbf{A})^{-1}\mathbf{1} = \beta\sum_{n=0}^{\infty}\alpha^n\mathbf{A}^n\mathbf{1}. \tag{1}$$

(d) Calculate the PageRank centrality

$$\mathbf{x} = \beta(\mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1})^{-1}\mathbf{1} = \beta\sum_{n=0}^{\infty}\alpha^n[\mathbf{A}\mathbf{D}^{-1}]^n\mathbf{1} \tag{2}$$

where $\mathbf{D}$ is a diagonal matrix with non-zero elements $D_{ii} = \kappa_i = \max(k_i^{out}, 1)$ and k_i^{out} is the out-degree of node i.

- **2. Degree centrality of a undirected network**
  Consider an undirected network with adjacency matrix $\mathbf{A}$.
  The degree centrality x_i of a node i is given by its degree k_i, i.e. $x_i = k_i$.
  Show that the degree centrality $\mathbf{x}$ of an undirected network satisfies the following equation

$$\mathbf{x} = \mathbf{A}\mathbf{D}^{-1}\mathbf{x}, \tag{3}$$

where $\mathbf{D}$ is a diagonal matrix with non-zero elements $D_{ii} = \kappa_i = \max(k_i, 1)$.