MTH5125: Actuarial Mathematics II

> **You should attempt ALL questions. Marks available are shown next to the questions.**

> **In completing this assessment:**
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> - **You may use books and notes.**
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> - **You may use calculators and computers, but you must show your working for any calculations you do.**
>
> - **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
>
> - **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;

- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;

- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. To be eligible for IFoA exemption, **you must submit your exam within the first 3 hours of the 24-hour exam period**.

Examiners: C.Sutton, L.Fang

Question 1 [20 marks]. A life assurance company issues a new Whole Life assurance policy to a life age 59 exact with a sum assured of £50,000 payable at the end of the policy year of death secured by annual premiums. In calculating these premiums the company assumes 4% per annum interest, that mortality follows the AM92 Ultimate table, and that expenses are 2.4% of each premium plus £120 to process a benefit claim.

(a) Write down an expression for the gross future loss random variable for this policy. [4]

(b) The life assurance company aims to issue new policies that have a 96% probability of making a profit. Calculate the minimum premium it should charge to meet this aim here. [6]

(c) The company is considering changing its profit target to be 2% of the sum assured for each new policy. Use an equation of value to calculate the annual premium it should charge on this basis. [6]

(d) What are the disadvantages of expressing a profit target as a percentage of sum assured? [4]

Question 2 [21 marks]. On 1st January 2020, Xenon Life issued 39 new Whole Life assurance policies to people age 62 exact. Fifteen of these policies have a sum assured of £82,000 and the remainder have a sum assured of £9,500. Premiums are annual and the sum assured is payable at the end of the policy year of death. Xenon Life calculates premiums and reserves on the following basis: Mortality AM92 Ultimate Interest 4% per annum Expenses 3.5% of each premium

(a) Calculate the office premium for these policies. [7]

(b) On 31st December 2021, Xenon Life pays death benefits totalling £19,000 in respect of these policies. Calculate the mortality profit for 2021. [10]

(c) Comment on the result in (b) above. [4]

Question 3 [18 marks]. Rowdy, now age 86, and Bash, who is exactly two years younger, were big rock 'n roll music stars in the 1950's. They have just sold the copyright to their music catalogue for £2 million and are interested in converting the proceeds into a continuous income stream. They approach an insurance company.

(a) Calculate the amount of annuity income per annum Rowdy and Bash would receive on a joint life basis payable continuously if the insurance company assumes 2% per annum interest, no expenses, and constant forces of mortality of 0.14 for Rowdy and 0.11 for Bash. **[9]**

(b) Rowdy's own financial advisor is worried that under the annuity in (a) above, should Bash die soon, Rowdy will not receive good value from the annuity purchase. They advise buying a reversionary annuity with the same annual amount again payable continuously, with Bash as counter-life. If the insurance company uses the same basis as in (a) above, what is the cost of this reversionary annuity? **[9]**

Question 4 [19 marks]. A life assurance company offers a 5-year term insurance product. For modelling purposes it assumes that premiums are payable continuously and ignores expenses. It assumes that the transition intensity related to mortality is a constant 0.00324.

(a) Draw and label a two-state model for this term insurance product. **[3]**

The company is considering adding a premium waiver benefit where if a policyholder becomes either unemployed or is diagnosed with a critical illness they will be allowed to cease paying premiums but continue to be eligible for the term insurance benefit. The company assumes that the unemployment transition intensity is 0.00892, the critical illness transition intensity is 0.00327, and that mortality rates are not affected by unemployment or critical illness.

(b) Draw and label a multi-state model which could be used to value the premium waiver benefit. **[4]**

(c) What percentage increase in premiums is needed once the premium waiver is introduced if the life assurance company is to maintain expected profit amounts in respect of the term insurance policies? **[12]**

Question 5 [22 marks]. A unit linked life assurance company offers a three year endowment where 99% of premiums are allocated to units with a bid-offer spread of 1.5% and an annual management fee of 2%. Expenses are assumed to be £39 per policy per annum. The company projects 7.5% per annum investment return in the unit fund and 2% per annum interest on cash balances. In assessing profitability of unit linked policies it uses a risk discount rate of 6% per annum.

(a) Construct projected revenue accounts for a policy where the premium is £2,000 per annum. [**10**]

The company is considering adding a guaranteed death benefit in the first year of the policy.

(b) Calculate the maximum guaranteed sum assured it can offer a 60 year old customer in the first policy year if it assumes AM92 Ultimate mortality rates and is willing to reduce the net present value of the policy by one-tenth. [**12**]

End of Paper.