Actuarial Mathematics II
MTH5125

Assessment 2 - setting for the excel file

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Model

Let $\mu_x^{01} = 10^{-5}$ and $\mu_x^{02} = A + Bc^x$ for all x where $A = 5 \times 10^{-4}$, $B = 7.4 \times 10^{-5}$ and $c = 1.05$.

Let $\tau = \max(5,\text{ the last digit of your student number })$. For example if your student number is 210473646 then $\tau = 6$.

Calculate:

(i) ${}_{\tau}p_{35}^{00}$ **[15 marks]**

(ii) ${}_{\tau}p_{35}^{02}$ **[15 marks]**

(iii) ${}_{\tau}p_{35}^{01}$ **[10 marks]** (a)-(i)

$$
\begin{aligned}
{}_{\tau}p_{35}^{00} &= \exp\left(-\int_0^{\tau}\left(\mu_{30+t}^{01}+\mu_{30+t}^{02}\right)dt\right) \\
&= \exp\left(-\int_0^{\tau}\left(10^{-5}+A+Bc^{x}\right)dt\right) \\
&= \exp\left(-10^{-5}\left(\tau-0\right)-\int_0^{\tau}\left(A+Bc^{35+t}\right)dt\right) \\
&= \exp\left(-10^{-5}\tau\right)\exp\left(-A\tau\right)\exp \\
&= \exp\left(-10^{-5}\tau\right)s^{\tau}g^{c^{35}\left(c^{\tau}-1\right)}
\end{aligned}
$$

where $s=\exp\left(-A\right)$, $g=\exp\left(-B/\log c\right)$

(a)-(ii)

$$ {}_{\tau}p_{35}^{02} = \int_{0}^{\tau} {}_{\tau}p_{35}^{00} \mu_{35+t}^{02} dt = \int_{0}^{\tau} {}_{\tau}p_{35}^{00} \left(A + Bc^{35+t}\right) dt $$

(a)-(iii)

$$ {}_{\tau}p_{30}^{01} = 1 -_{\tau} p_{30}^{00} -_{\tau} p_{30}^{02} $$

An insurance company uses the model above to calculate premiums for a special τ-year term life insurance policy. The basic sum insured is \$100,000, but the death benefit is \$150,000 if death occurs as a result of an accident. The death benefit is payable immediately on death. Premiums are payable continuously throughout the term. The effective rate of interest is 3% per year and there are no expenses. The policy is issued to a life aged 35.

(i) calculate the annual premium for this policy **[20 marks]**

(ii) calculate the policy value at time $1, 2, \tau - 2, \tau - 1$ and τ in state 0 **[25 marks]**

(iii) comment on the results at (ii) **[15 marks]**

(b) -(i) The EPV for the premium P per year payable continuously is:

$$P\bar{a}^{00}_{35:\overline{\tau}|} = P\int_0^{\tau} v^t \; {}_tp^{00}_{35} dt$$

The EPV of the death benefit is:

$$\begin{aligned} & 150,000\bar{A}^{01}_{35:\overline{\tau}|} + 100,000\bar{A}^{02}_{35:\overline{\tau}|} \\ = \; & 150,000\int_0^{\tau} v^t \; {}_tp^{00}_{30}\mu^{01}_{30+t} dt + 100,000\int_0^{\tau} v^t \; {}_tp^{00}_{30}\mu^{02}_{30+t} dt \end{aligned}$$

$$P = \frac{150,000\bar{A}^{01}_{35:\overline{\tau}|} + 100,000\bar{A}^{02}_{35:\overline{\tau}|}}{\bar{a}^{00}_{35:\overline{\tau}|}}$$

The integrals are evaluated using numerical integration.

The policy value at time 5 in state 0 is given by:

$$ {}_1V^0 = 100,000\bar{A}^{02}_{36:\overline{\tau-1|}} + 150,000\bar{A}^{01}_{36:\overline{\tau-1|}} - P\bar{a}^{00}_{36:\overline{\tau-1|}} $$

$$ {}_2V^0 = 100,000\bar{A}^{02}_{37:\overline{\tau-2|}} + 150,000\bar{A}^{01}_{37:\overline{\tau-2|}} - P\bar{a}^{00}_{37:\overline{\tau-2|}} $$

$$ {}_{\tau-2}V^0 = 100,000\bar{A}^{02}_{35+\tau-2:\overline{2|}} + 150,000\bar{A}^{01}_{35+\tau-2:\overline{2|}} - P\bar{a}^{00}_{35+\tau-2:\overline{2|}} $$

$$ {}_{\tau-1}V^0 = 100,000\bar{A}^{02}_{35+\tau-1:\overline{1|}} + 150,000\bar{A}^{01}_{35+\tau-1:\overline{1|}} - P\bar{a}^{00}_{35+\tau-1:\overline{1|}} $$

$$ {}_{\tau}V^0 = 0 $$