Actuarial Mathematics II
MTH5125

Whole life annuities with m-thly payments

Dr. Melania Nica

Spring Term

Life Annuities with m-thly Payments

In practice, life annuities are often payable on a monthly, quarterly, or semiannual basis.
An m-thly life annuity-due makes a payment of $\frac{1}{m}$ at the beginning of every m-thly period so that in one year the total payment is 1.
First we will need to find the whole life insurance benefit paid at the end of m-thly period.

Remember interest rates

Reminder:

Interest factor: $(1+i)^{1/m} = 1 + \frac{i^{(m)}}{m}$

Effective interest rate: $\frac{i^{(m)}}{m} = (1+i)^{1/m} - 1 = v^{-1/m} - 1$

The discount factor: $(1+i)^{-1/m} = (1-d)^{-1/m} = 1 - \frac{d^{(m)}}{m}$

The effective discount rate: $\frac{d^{(m)}}{m} = 1 - v^{1/m}$

Life Annuities with m-thly Payments

$$\begin{aligned}\ddot{a}_x^{(m)} &= \frac{i^{(m)} - i\left(1 - d\ddot{a}_x\right)}{i^{(m)} d^{(m)}} \\ &= \frac{id}{i^{(m)} d^{(m)}} \ddot{a}_x - \frac{i - i^m}{i^{(m)} d^{(m)}}\end{aligned}$$

$$\ddot{a}_x^{(m)} = \alpha\left(m\right) \ddot{a}_x - \beta\left(m\right)$$

$\alpha\left(m\right)$ approximated to 1
$\beta\left(m\right)$ approximated to $\frac{m-1}{2m}$

Term Life Annuities with m-thly Payments

$$
\begin{aligned}
\ddot{a}^{(m)}_{x:\overline{n}|} &= \ddot{a}^{(m)}_{x} - v^{n}{}_{n}p_{x}\,\ddot{a}^{(m)}_{x+n} \\
&= \alpha(m)\ddot{a}_{x} - \beta(m) - v^{n}{}_{n}p_{x}\ \left(\alpha(m)\ddot{a}_{x+n} - \beta(m)\right) \\
&= \alpha(m)\left(\ddot{a}_{x} - v^{n}{}_{n}p_{x}\,\ddot{a}_{x+n}\right) - \beta(m)\left(1 - v^{n}{}_{n}p_{x}\right) \\
&= \alpha(m)\,\ddot{a}_{x:\overline{n}|} - \beta(m)\left(1 - v^{n}{}_{n}p_{x}\right)\ .
\end{aligned}
$$