Actuarial Mathematics II
MTH5125

Revision: Life Tables
Chapter 3 (DHW)

Dr. Melania Nica

Spring Term

Life Tables

- A **survival model** is set of forces-of-mortality

$$\{\mu_y \mid y \geq x_0\}$$

used to determine the survival probabilities of a specified group of persons.

- For any person (x), $x \geq x_0$, belonging to this specified group, μ_y, $y \geq x$, is his assumed force-of-mortality at age y.
- Example of a survival model: **Makeham survival model**:

$$\mu_y = A + Bc^y, \qquad \text{for all } y \geq x_0$$

- Examples of a specified group:
  - All male smokers of age $x \geq x_0$ who underwrite this year a life insurance contract with insurer A.
  - All female persons of age $x = x_0$ who underwrite this year a lifelong pension contract with insurer B.

Life tables

- Consider the following *survival model*:

$$\boxed{\{\mu_y \mid y \geq x_0\}}$$

- Survival probabilities for (x) whose survival probabilities follow from this model:

$${}_t p_{x+u} = \exp\left(-\int_0^t \mu_{x+u+s}\, ds\right), \qquad x \geq x_0 \text{ and } u \geq 0$$

- **Life table** constructed from this survival model:
  - l_{x_0} = arbitrary positive number, called the **radix**.
  - For $t \geq 0$, define l_{x_0+t} by

$$\boxed{l_{x_0+t} = l_{x_0} \times {}_t p_{x_0}}$$

Life tables

- Consider (x), $x \geq x_0$, who follows the survival model $\{\mu_y \mid y \geq x_0\}$, with corresponding life table $\{l_y \mid y \geq x_0\}$.
- Survival probabilities for (x):

$$\boxed{{}_t p_x = \frac{l_{x+t}}{l_x}} \quad \text{for any } t \geq 0 \tag{3.1}$$

- Notation:

$$d_x \stackrel{\text{not.}}{=} l_x - l_{x+1} \tag{3.4}$$

- One-year mortality rates:

$$\boxed{q_x = \frac{d_x}{l_x}} \tag{3.5}$$

- Deferred mortality rates:

$$\boxed{{}_{t|u} q_x = \frac{l_{x+t} - l_{x+t+u}}{l_x}}$$

Life tables

- Interpretation of l_{x+t}:
  - Let $\mathbf{L}_{x+t}$ be the number of survivors at age $x+t$ from a closed group of l_x persons of age x, with survival probabilities following from the survival model $\{\mu_y \mid y \geq x_0\}$.
  - The expected number of survivors:

$$\boxed{\mathbb{E}[\mathbf{L}_{x+t}] = l_{x+t}}$$

- Interpretation of d_x:
  - Let $\mathbf{D}_x$ be the number of deaths in the year of age x to $x+1$ from the same closed group of l_x persons of age x.
  - The expected number of deaths:

$$\boxed{\mathbb{E}[\mathbf{D}_x] = d_x}$$

Life tables

- Relations:

$$\boxed{{}_tp_x = \frac{l_{x+t}}{l_x}}, \quad \boxed{q_x = \frac{d_x}{l_x}} \quad \text{and} \quad \boxed{{}_{t|u}q_x = \frac{l_{x+t} - l_{x+t+u}}{l_x}}$$

- Stochastic interpretation:
  - p - and q - functions are *probabilities*.
  - l - and d - functions are *expected numbers* of survivors and dyers from a closed group of l_x persons.
- Deterministic interpretation:
  - p - and q - functions are *fractions*.
  - l - and d - functions are *observed numbers* of survivors and dyers from a closed group of l_x persons.