Queen Mary University of London

MTH786U/P, Semester A, 2023/24

Coursework 0 **N. Perra**

Problem 1

1. Compute the gradient ∇L of the function $L : \mathbb{R}^2_+ \to \mathbb{R}$ defined as

$$L(x, y) = \frac{x}{y} - 1 - \log\left(\frac{x}{y}\right) .$$

   Here $\mathbb{R}^2_+$ is the space of all real two-dimensional vectors with positive entries.

2. Show that L from Question 1 is scalar-invariant, i.e. $L(x, y) = L(c\,x, c\,y)$ for any scalar $c > 0$ and all arguments $x > 0, y > 0$.

Problem 2

1. Compute the expected value $\mathbb{E}_x$ of a (discrete) Poisson-distributed random variable X with probability

$$\rho_x := \exp(-\lambda)\frac{\lambda^x}{x\,!}\,, \quad x = 0, 1, 2, \ldots, s$$

   for a constant $\lambda > 0$. What is the solution for $s \to \infty$?
   Hint: Make use of the identity $\exp(\lambda) = \sum_{x=0}^{\infty} \frac{\lambda^x}{x\,!}$.

2. For a uniform (and absolutely continuous) random variable X in $[0, 1]$ compute the expectation of $f(X)$ for

$$f(x) := \begin{cases} -\log(x) & x \in [0, 1/5] \\ 0 & \text{otherwise} \end{cases} ,$$

   Make use of the convention $0 \log(0) = 0$.

Problem 3

1. Let X be a random variable with expectation μ and variance σ^2. Show that the variance of $aX + b$, where $a, b \in \mathbb{R}$, is

$$\mathrm{Var}_x[ax + b] = a^2\sigma^2 .$$