MTH6105 – Algorithmic Graph Theory **Spring 2024**

Problem Sheet 1 **F. Fischer**

You are expected to **attempt all exercises** before the seminar and to **actively participate** in the seminar itself.

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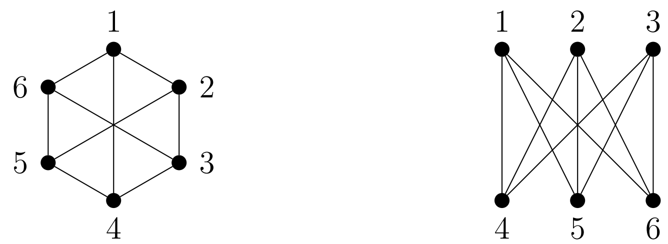
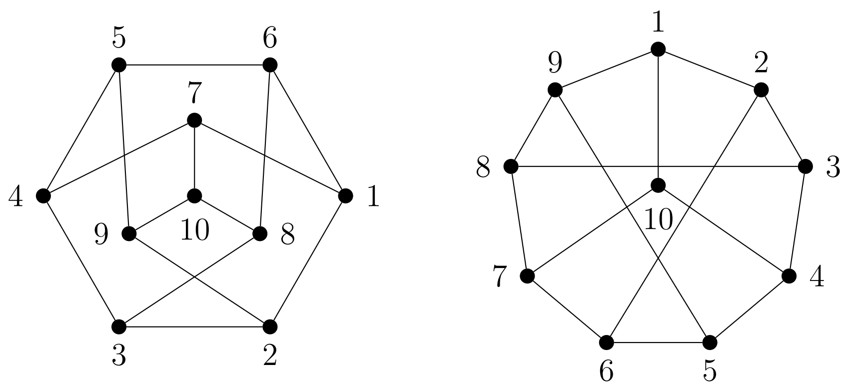
1. For each pair of graphs shown below, either show that they are isomorphic by giving a bijection or explain why they are not isomorphic.

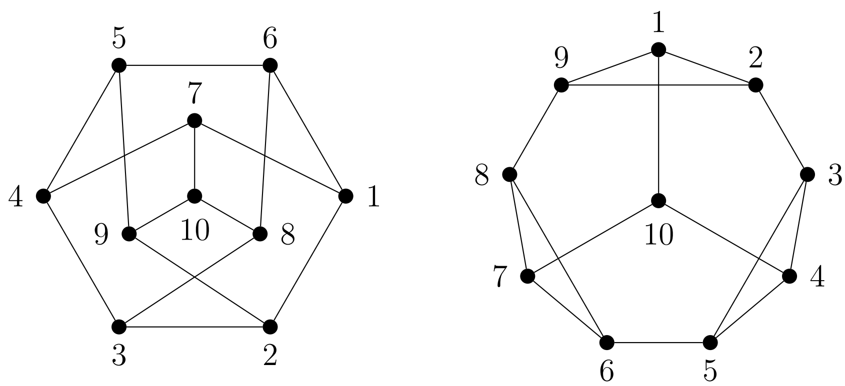
(a)



(b)



(c)



Note: We have defined which properties a bijection must possess to show that two graphs are isomorphic, but have not discussed an algorithm for finding such a bijection. You may therefore have to look for a bijection by trial and error, and you will probably conclude that this is a bit tedious. To show that two graphs are *not* isomorphic, you have to argue that a bijection with the desired property does not exist. This is again likely to involve some trial and error.

2. Define the complement of a simple graph G as the graph G^c with $V(G^c) = V(G)$ such that for all distinct $u, v \in V(G^c)$, $uv \in E(G^c)$ if and only if $uv \notin E(G)$. Consider a simple graph G with $|V(G)| \geq 6$.

   (a) Show that G or G^c must contain a vertex of degree at least 3.

   (b) Show that G or G^c must contain a subgraph that is isomorphic to C_3. To this end, you may want to consider the adjacencies among neighbors of a vertex of degree 3.

   (c) Show that in any group of at least 6 people, there are always 3 that mutually know each other, or 3 that mutually do not know each other.

3. Show that isomorphism forms an equivalence relation on the set of simple graphs. To do this, you must show each of the following:

   (a) (symmetry) For any simple graphs G and H, if G is isomorphic to H, then H is isomorphic to G.

   (b) (reflexivity) For any simple graph G, G is isomorphic to G.

   (c) (transitivity) For any simple graphs F, G, and H, if F is isomorphic to G and G is isomorphic to H, then F is isomorphic to H.