MTH6105 – Algorithmic Graph Theory **Spring 2024**

Assessed Coursework 2 **F. Fischer**

This assessment consists of three exercises, which carry equal weight and together contribute **10% of your mark for the module**. Please **upload your answers before the deadline**.

Any work you submit must be your own. You may discuss the exercises with other students, but you must write up your solution yourself. Copying a solution or submitting someone else's solution constitutes an assessment offence.

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1. Consider the network (G, w) with

$$V(G) = \{a, b, c, d, e, f\},$$
$$E(G) = \{ab, ac, ad, ae, af, bc, bd, be, bf, cd, ce, cf, de, df, ef\},$$

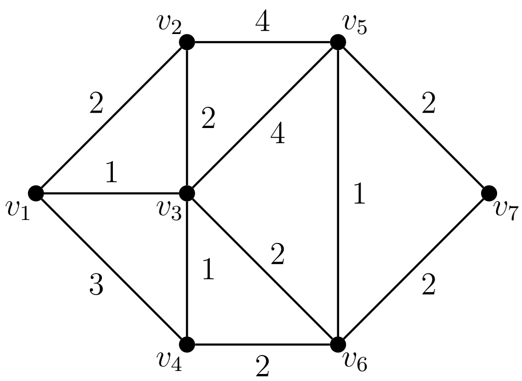
and

$$w(ab) = 9, \quad w(ac) = 8, \quad w(ad) = 12, \quad w(ae) = 3, \quad w(af) = 15,$$
$$w(bc) = 5, \quad w(bd) = 6, \quad w(be) = 13, \quad w(bf) = 10, \quad w(cd) = 4,$$
$$w(ce) = 14, \quad w(cf) = 2, \quad w(de) = 16, \quad w(df) = 11, \quad w(ef) = 7.$$

(a) Use Kruskal's algorithm to find a minimum spanning tree of (G, w). List the edges of the tree in the order in which they are added, and draw the tree.

(b) Show that the minimum spanning tree found by Kruskal's algorithms is in fact the unique minimum spanning tree of (G, w).

2. Consider the following network (G, w).



(a) Use Dijkstra's algorithm to find a shortest v_1–v_7-path. Give $V(T)$ and $E(T)$ after each iteration of the algorithm.

(b) Explain the effect each of the following changes of the length of a single edge would have on the length of a shortest v_1–v_7-path:

(i) an increase of $w(v_3v_4)$ from 1 to 2;

(ii) a decrease of $w(v_6v_7)$ from 2 to 1;

(iii) a decrease of $w(v_3v_5)$ from 4 to 1.

3. Consider a network (G, w). Let $c \in \mathbb{R}$, and let $m : E(G) \to \mathbb{R}$ such that $m(e) = w(e) + c$ for all $e \in E(G)$.

   (a) Show that T is a minimum spanning tree of (G, w) if and only if it is a minimum spanning tree of (G, m).

   (b) What is the analogous claim for shortest paths in (G, w) and (G, m)? Prove this claim or provide a counterexample showing that it is not true.