QUEEN MARY, UNIVERSITY OF LONDON
MTH6102: Bayesian Statistical Methods

Exercise sheet 1

2023-2024

Note: there is a table of common distributions on the module's QMPlus page.

1. We want to estimate q, the proportion of people in a population who have a certain illness. We test 200 people, and find that 18 of them have the illness.

   Find the MLE $\hat{q}$ for q. What is the standard error of $\hat{q}$?

2. The survival time of a certain type of mosquito in days following adult emergence is assumed to follow an exponential distribution with parameter λ. We observe 6 mosquitoes, and see lifetimes $(4, 12, 6, 19, 9, 5)$.

   What is the MLE for λ?

   Suppose that the data was as above, except that instead of dying after 19 days, the fourth mosquito was observed to be still alive after 20 days, with the exact time of death not known. Now what is the MLE for λ?

3. Consider the simple linear regression model where $Y_1, \ldots, Y_n$ are independent random variables with
   $$Y_i \sim N(\mu_i, \sigma^2), \ \mu_i = \beta_0 + \beta_1 x_i, \ \text{for } i = 1, \ldots, n.$$

   Given that we observe $y_1, \ldots, y_n$, find the MLEs for β_0, β_1 and σ^2.

   (Hint: for $\hat{\beta}_0$ and $\hat{\beta}_1$, look in your notes for Statistical Modelling 1, and compare the log-likelihood that we are using to the sum of squares that was used in that module. Then the MLEs $\hat{\beta}_0$ and $\hat{\beta}_1$ can be used to find the MLE for σ.)