MTH6102: Bayesian Statistical Methods

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MLE: Normal example

- $Y_1, \ldots, Y_n$ be an i.i.d $\sim N(\mu, \sigma^2)$.
- There are two unknown parameters.
- So θ is a vector, $\theta = (\mu, \sigma)$.
- **Exercise:** Find the likelihood function and the MLE of θ.

Assessing uncertainty

- Suppose we want to estimate a population parameter θ.
- In frequentist statistics the idea is to design an estimator $\hat{\theta}$, where an estimator is a function of the data.
- Sample statistics or estimators vary from sample to sample (they will not match the parameter exactly)
- We usually want to assess the uncertainty in any estimate.

Assessing uncertainty

- **KEY QUESTIONS**: For a given sample statistic, what are plausible values for the population parameter? How much uncertainty surrounds the sample statistic?
- **KEY ANSWER:** It depends on how much the statistic varies from sample to sample!
- In frequentist statistics, two common summaries of the uncertainty are:
  - the standard error;
  - a confidence interval.
- The quantify the uncertainty in $\hat{\theta}$ due to random variation in the data we might have observed.

Sampling distribution

- Frequentist statistics uses the idea of the sampling distribution.
- If we could repeatedly generate data from a certain model, we would get a distribution of values for $\hat{\theta}$.
- This is the sampling distribution for $\hat{\theta}$.

A sampling distribution is the distribution of sample statistics computed for different samples of the same size from the same population.

- A sampling distribution shows us how the sample statistic varies from sample to sample

Standard error

The standard error of $\hat{\theta}$ is the standard deviation of the sampling distribution

- It quantifies the spread or the variability of the sampling distribution.
- So this is the simplest summary of the uncertainty in $\hat{\theta}$.
- It measures how much the statistic varies from sample to sample and quantifies the uncertainty in $\hat{\theta}$ due to random variation in the data we might have observed.

History of likelihood

- The use of likelihood in frequentist statistics was mainly developed by Ronald Fisher.
- "On an Absolute Criterion for Fitting Frequency Curves"
- Published in 1912, while he was a maths undergraduate.
- Later papers more fully developed the theory.