PROBLEM SET 3 FOR MTH 6151

1. Find the general solution to the pde

$$x^2U_x + y^2U_y = (x+y)U.$$

2. Use the method of characteristics to solve the problem

$$\begin{aligned} &yU_x + xU_y = 0, \\ &U(0,y) = e^{-y^2}. \end{aligned}$$

3. Use the method of characteristics to solve the problem

$$\begin{aligned} &U_x + U_y + U = e^{x+2y}, \\ &U(x,0) = 0. \end{aligned}$$

4. Solve the problem

$$\begin{aligned} &N_x + N_t = -\mu N, \\ &N(0,t) = e^{-t}, \end{aligned}$$

where $\mu > 0$ is a positive constant.

5. In class we saw that the characteristic curves of the equation

$$U_x + yU_y = 0$$

are given by the equation

$$y(x) = Ce^x, \qquad C \in \mathbb{R}.$$

Show that the above family of curves fills the whole plane $\mathbb{R}^2$. HINT: this means that for a given point $(x_\star, y_\star)$, one can always find one and only one characteristic curve passing through the point.

6. Find the general solutions to the PDE

$$U_x - U_t = \frac{1}{U^2}.$$

7. Solve the initial value problem

$$\begin{cases} U_x + 2U_t = e^{-U}, t \geq 0 \\ U(x,0) = 1 \end{cases}$$

8. Classify, according to type (hyperbolic, elliptic, parabolic) the equations

(i) $U_{xx} + 2U_{xy} + U_{yy} - U_x + U = 0.$
(ii) $2U_{xy} + U_y + U_x = 0.$
(iii) $U_{xx} - U_{xy} - 2U_{yy} = 0.$
(iv) $2U_{xx} + 4U_{xy} + 3U_{yy} - 5U = 0.$

9. Transform the following equations to a canonical form (i.e. without cross-derivatives):

(i) $4U_{xx} + 12U_{xy} + 9U_{yy} = 0.$
(ii) $2U_{xx} - 4U_{xy} - 6U_{yy} + U_x = 0.$
(iii) $U_{xx} + 2U_{xy} + 17U_{yy} = 0.$

To which type do each of the equations above belongs to?